

SAT Math Workbook

Volume 1

Triumph College Admissions



Skill Lesson

Quiz Time!

A B C ☒ E



Hints!

A B C ☒ E



Answers!

A B C ☒ E



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Math Lesson #1

Basic Math Skills

- *Arithmetic*
- *Divisibility*
- *Ratios*
- *Probability*

Triumph College Admissions

 **Skill Lesson**

Quiz Time! 
☐ A ☐ B ☐ C ☒ D ☐ E

Hints! 
☐ A ☐ B ☐ C ☒ D ☐ E

Answers! 
☐ A ☐ B ☐ C ☒ D ☐ E

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ARITHMETIC

This skill refers to questions that can be solved with addition, subtraction, multiplication, or division.

Because you can use a calculator on the SAT, exam questions will not be pure arithmetic operations. Nevertheless, don't forget how to do order of operations. (Remember to work in **P**arentheses, simplify **E**xponents, work **M**ultiplication and **D**ivision in order from left to right, work **A**ddition and **S**ubtraction in order from left to right. **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally.

There are few SAT questions that require only simple arithmetic to be solved; most "arithmetic" questions have multiple steps and test other skills as well. The questions are often in the form of word problems and you have to decide when to add, subtract, multiply, and divide.

Just keep some basics in mind. In general, if you are given a total amount, then you should either subtract or divide. If you are asked to find the total amount, then you should either add or multiply.

Here are some examples to help you develop your "arithmetic" skills.

Example 1:

How many cartons of eggs, each holding 1 dozen, are needed to hold 300 eggs? (One dozen equals 12.)

Hint:

Think: Will the answer be larger or smaller than 300?

Solution:

The answer must be smaller than 300. You should either subtract or divide. Because 12 eggs are in 1 dozen, you need to find out how many dozen are in 300. You divide 300 by 12. The answer is 25.

Example 2:

Each student in a class sold 4 cakes to raise money for the class. If each student charged \$2.50 per cake, and a total of \$300 was raised, how many students are in the class?

Hint:

Make sure you answer the question that is asked and not what you assume will be asked. The question could have ended with how much did each student earn?

Solution:

Each student raised $4 \times \$2.50 = \10

If a total of \$300 was raised, then there must be $\frac{\$300}{\$10} = 30$ students.

The following two examples use units with which you are familiar. The way you solve these problems will help you with similar problems where the units make no sense.

Example 3:

How many seconds are in 5 hours?

Hint:

How many seconds are in a minute? How many minutes are in an hour?

Solution:

1 hour = 60 minutes

1 minute = 60 seconds

So substitute...

1 hour = $60(\underline{60 \text{ seconds}}) = \underline{3,600 \text{ seconds}}$

5 hours = $5(3,600) = 18,000 \text{ seconds}$

Example 4:

Through how many degrees does a minute hand of a clock move in 40 minutes?

Hint:

One way to solve this type of problem is to figure how many degrees a minute hand moves in 1 minute.

Solution:

A minute hand moves 6 degrees in 1 minute. (Divide 360 degrees in one revolution by 60 minutes in one revolution.) The answer is then $40 \times 6 = 240$.

Example 5:

Through how many degrees does an hour hand of a clock move in an hour? A half hour? A quarter of an hour?

Hint:

How many degrees are in one revolution?
How many hours are in one revolution?

Solution:

An hour hand moves 30 degrees in 1 hour. (This is similar to Example 4, but in this case 360 degrees in one revolution is divided by 12 hours per revolution.)
One hour = 30 degrees. One-half hour = 15 degrees. One-quarter hour = 7.5 degrees.

Studyguide for the SAT Skill Review: Arithmetic

On some problems you have to be concerned with place values of the numbers. The following problem is a refresher for place values.

Example 6:

A four-digit number has A as the thousands digit, B as the hundreds digit, C as the tens digit, and D as the units digit. Which of the following is a representation of the number?

- (A) $100A + 10B + C + D$
- (B) $1000A + 100B + 10D + C$
- (C) $10^3A + 10^2B + 10C + D$
- (D) $10^3A + 10^2B + 10^0D + C$

Hint:

Remember that 10 to the power of 1 is 10 and 10 to the power of 0 is 1.

Solution:

You can write the answer two ways, $1000A + 100B + 10C + D$ or using exponents:

$10^3A + 10^2B + 10C + D$. The answer is (C).

Students often skip the following type of problem because of the letters in them. **Don't let the letters bother you.**

Example 7:

What is the sum of A and B in the correctly worked addition problem below?

$$\begin{array}{r} 7A \\ + B6 \\ \hline 159 \end{array}$$

Hint:

Take the time to substitute numbers for **A** and **B** until you find numbers that work.

Solution:

A must be 3 because $\underline{3} + 6 = 9$ and **B** must be 8 because $7 + \underline{8} = 15$.
Just remember the solution is the SUM of **A** and **B**. Answer = 11 (3 + 8).



Grid-in your answer here:

Question 1

How many bottles, each holding 12 fluid ounces, are needed to hold 6 quarts of orange juice? (1 quart equals 32 fluid ounces.)

.	/	/	.	.
	0	0	0	
1	1	1	1	
2	2	2	2	
3	3	3	3	
4	4	4	4	
5	5	5	5	
6	6	6	6	
7	7	7	7	
8	8	8	8	
9	9	9	9	

Question 2

How many more bags would be needed to package 1,600 marbles in bags of 10 than in bags of 16?

- (A) 6
(B) 10
(C) 60
(D) 100
(E) 600

Question 3

In the correctly worked addition problem below, each X represents the same digit. What is the value of X?

$$\begin{array}{r} \text{X5} \\ \text{X7} \\ \text{X8} \\ +\text{X9} \\ \hline 229 \end{array}$$

- (A) 2
(B) 3
(C) 4
(D) 5
(E) 6

Studyguide for the SAT Skill Quiz A: Arithmetic

Question 4

Which of the following numbers has the digit 5 in the thousandths place?

- (A) .0005
- (B) .0050
- (C) .050
- (D) .5000
- (E) 5,000.0

Question 5

Which of the following is equal to 14?

- (A) $2 \times [8 - (6 + 3)]$
- (B) $2 \times [(8 - 6) / 3]$
- (C) $[2 \times (8 - 6)] / 3$
- (D) $[(2 \times 8) - 6] / 3$
- (E) $(2 \times 8) - (6 / 3)$

Question 6

If $x + 5$ is an even integer, then x could be any of the following, except...

- (A) -5
- (B) -2
- (C) 1
- (D) 3
- (E) 5

Question 7

Which of the following numbers has the digit 6 in the hundreds place?

- (A) $5 \times 10^3 + 4 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$
- (B) $6 \times 10^3 + 5 \times 10^2 + 4 \times 10^1 + 3 \times 10^0$
- (C) $4 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$
- (D) $3 \times 10^3 + 6 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$
- (E) $5 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 6 \times 10^0$

Question 8

Through how many degrees does the minute hand of a clock turn from 7:05 a.m. to 7:25 a.m. of the same day?

- (A) 20
- (B) 90
- (C) 120
- (D) 180
- (E) 240

Grid-in your answer here:

Question 9

On a certain map a distance of 15 miles is represented by 1.0 centimeter. How many miles are represented by 4.5 centimeters on the map?

	/	/	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Question 10

If A, B, and C are the hundreds, tens, and units digits, respectively, of a number, how many tens are equal to that number?

- (A) $A + \frac{B+C}{10}$
- (B) $A + \frac{10B+C}{10}$
- (C) $\frac{A}{10} + B + 10C$
- (D) $10A + B + \frac{C}{10}$
- (E) $100A + 10B + C$

Studyguide for the SAT Skill Quiz A: Arithmetic



Hint for Question 1:

How many total ounces of orange juice need to be bottled?
Should the answer be larger or smaller than the total ounces?

Hint for Question 2:

How many bags of 10 marbles are needed?
How many bags of 16 marbles are needed?

Hint for Question 3:

Here is a big hint for this type of problem. You can solve this by just substituting in answers. Note that answers will always be in order either smallest to largest or largest to smallest. If answers are smallest to largest, start by substituting the (C) answer. If (C) is too big, go to (A) or (B). If (C) is too small, go to (D) or (E).

Hint for Question 4:

What are the place-values to the right of the decimal?

Hint for Question 5:

Use order of operations. Your calculator **might not** follow order of operations when doing arithmetic calculations.

Hint for Question 6:

One way to solve is by substituting numbers.
What kind of number are you looking for?

Hint for Question 7:

Ten to what power is 100?

Hint for Question 8:

How many minutes are from 7:05 a.m. to 7:25 a.m., and how many degrees are in one minute?

Hint for Question 9:

Substitute the number of miles for 1 centimeter.

Hint for Question 10:

Write the number using place-values. Then think how you would answer a similar questions using numbers. For example: How many tens are equal to 140? What do you have to do to answer the question?



Question 1.

The answer is 16.

Explanation for Question 1:

The total ounces of orange juice is $6 \times 32 = 192$. To find the number of bottles divide 192 by 12. $192 \div 12 = 16$. The answer is 16.

Question 2.

The answer is (C).

Explanation for Question 2:

The number of bags of 10 marbles is 160. The number of bags of 16 marbles is 100. To find how many more bags for 10 marbles than for 16 marbles you subtract $160 - 100 = 60$. The answer is (C).

Question 3.

The answer is (D).

Explanation for Question 3:

You can find the answer to this problem by using backsolving. Choose one of the answers and substitute it for X. If you substitute answer (C), $X = 4$, into the problem your sum is 189. Because this sum is too small try an answer that will give a larger sum. Try either answer (D) or (E). If you try answer (D), $X = 5$, the sum will be 229. This is the sum we wanted. The answer is (D).

Question 4.

The answer is (B).

Explanation for Question 4:

To the right of the decimal are place-values in order: tenths, hundredths, thousandths, ten-thousandths, etc. The answer is (B).

Question 5.

The answer is (E).

Explanation for Question 5:

Remember to do the operations inside the brackets and parentheses first.

$$(2 \times 8) - (6/3) = 16 - 2 = 14$$

The answer is (E).

Studyguide for the SAT Skill Quiz A: Arithmetic

Question 6.

The answer is (B).

Explanation for Question 6:

The only value that gives an odd answer is -2. The answer is (B).

Question 7.

The answer is (D).

Explanation for Question 7:

10^2 is 100. The answer is (D).

Question 8.

The answer is (C).

Explanation for Question 8:

Multiply 20 minutes by 6 degrees per minute to get 120 degrees. The answer is (C).

Question 9.

The answer is 67.5.

Explanation for Question 9:

15 miles = 1.0 centimeter
Answer = 4.5 centimeters
Answer = 4.5 (15 miles)
Answer = 67.5 miles
The answer is 67.5.

Question 10.

The answer is (D).

Explanation for Question 10:

Writing the number in place-value you get $100A + 10B + C$. You must divide this number by 10 to find out how many 10's are equal to it.

$$\frac{(100A + 10B + C)}{10} = 10A + B + \frac{C}{10}$$

(Remember you have to divide each term in the numerator by 10.)

The answer is (D).

An alternative method is to multiply each answer by 10 to get $100A + 10B + C$. Just remember to distribute the 10 correctly.



Question 1

Peggy received pledges from 40 people for a 6-mile walk-a-thon. Peggy walked 6 miles and each person gave \$0.25 for each mile she walked. Which of the following gives the total dollar amount Peggy collected?

- (A) $6 \times 0.25 + 40$
- (B) $6 \times 0.25 \times 40$
- (C) $40 \times 6 + 0.25$
- (D) $6 + 40 \times 0.25$
- (E) $6 + 40 + 0.25$

Grid-in your answer here:

Question 2

Packs of gum are 3 for \$0.70, and baseball cards are 4 for \$1.12. What is Cato's change from \$5.00 if he buys six packs of gum and 12 baseball cards?

	/	/	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Question 3

If an object travels at 4 feet per minute, how far does it travel in one-quarter hour?

- (A) 1
- (B) 4.25
- (C) 15
- (D) 44
- (E) 60

Question 4

11:30 a.m. is how many minutes past 9:45 a.m.?

- (A) 45
- (B) 65
- (C) 90
- (D) 105
- (E) 125

Question 5

The number $a - 5$ is how much less than $a + 5$?

- (A) 5
- (B) 10
- (C) $a - 10$
- (D) $a - 5$
- (E) $2a$

Question 6

If an "octaminute" is equivalent to 8 minutes of time, how many octaminutes are equivalent to 4 hours of time?

- (A) 30
- (B) 32
- (C) 240
- (D) 480
- (E) 1,920

Question 7

If flour costs p cents a pound, how many pounds of flour can be bought for \$3.00?

- | | |
|---------------------|---------------------|
| (A) $300p$ | (D) $\frac{3}{p}$ |
| (B) $3p$ | |
| (C) $\frac{300}{p}$ | (E) $\frac{p}{300}$ |

Studyguide for the SAT Skill Quiz B: Arithmetic

Question 8

A certain building has 3,500 square feet of surface that needs to be painted. If 1 gallon of paint will cover 300 square feet, what is the least whole number of gallons that must be purchased in order to have enough paint to apply one coat to the surface? (Assume that only whole gallons of paint can be purchased.)

- (A) 11
(B) 12
(C) 13
(D) 20
(E) 32

Question 9

Jerry spends \$2.95 for lunch at school each day. He wants to estimate the amount he will spend for lunch during the month of December, which has 18 school days. Which of the following will give him the best estimate?

- (A) 2.00×15
(B) 2.00×20
(C) 2.50×10
(D) 2.50×15
(E) 3.00×20

Grid-in your answer here:

Question 10

$$\begin{array}{r} 9\text{B}3 \\ \text{A}65 \\ +\underline{39\text{C}} \\ \hline \text{B}872 \end{array}$$

In the above problem B, C, and A represent digits in the correctly worked addition problem.

What is the sum of B, C, and A?

.	/	/		
.
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Studyguide for the SAT Skill Quiz B: Arithmetic



Hint for Question 1:

Look at the answer choices. You don't have to get an actual number. What operation should you use?

Hint for Question 2:

What would six packs of gum cost? What would 12 baseball cards cost? What would the total cost be?

Hint for Question 3:

How many minutes are in one-quarter hour? What operation do you use?

Hint for Question 4:

Review how to subtract times. Don't forget to borrow 1 hour (60 minutes).

Hint for Question 5:

Which expression do you subtract from the other?

Hint for Question 6:

How many minutes are in 4 hours? What operation do you use?

Hint for Question 7:

One technique is to make up a price and figure out what operation to use. However, before you begin be sure to change the units to cents. For example: if flour costs 3 cents per pound, how many pounds could be bought for \$3.00? Use the same operation with the variable p to answer the question.

Hint for Question 8:

What operation would you use?

Hint for Question 9:

You don't need to do any multiplication to get this answer. In which expression are the numbers closest to \$2.95 and 18?

Hint for Question 10:

Substitute numbers until you find ones that work.

Studyguide for the SAT Skill Quiz B: Arithmetic



Question 1.

The answer is (B).

Explanation for Question 1:

You should multiply to find the answer. The answer is (B).

Question 2.

The answer is .24.

Explanation for Question 2:

Six packs of gum would cost $2 \times \$0.70 = \1.40 , and 12 baseball cards would cost $3 \times \$1.12 = \3.36 . The total cost would then be $\$1.40 + \$3.36 = \$4.76$. The change from \$5.00 is then $\$5.00 - \$4.76 = \$0.24$. The answer is .24.

Question 3.

The answer is (E).

Explanation for Question 3:

There are $\frac{60}{4} = 15$ minutes in one-quarter hour. You multiply $15 \times 4 = 60$. The answer is (E).

Question 4.

The answer is (D).

Explanation for Question 4:

One way to solve the problem is to subtract the beginning time from the ending time. Borrow 1 hour (60 minutes from 11) and add to the 30.

$$\begin{array}{r} 11:30 \\ -9:45 \\ \hline \end{array} \qquad \begin{array}{r} 10:90 \\ -9:45 \\ \hline 1:45 = 105 \text{ minutes} \end{array}$$

The answer is (D).

Question 5.

The answer is (B).

Explanation for Question 5:

Subtract $a - 5$ from $a + 5$. Don't forget to distribute the - !

$$\begin{array}{r} a + 5 \\ -(a - 5) \\ \hline \end{array} \qquad \begin{array}{r} a + 5 \\ -a + 5 \\ \hline 10 \end{array}$$

The answer is (B).

Studyguide for the SAT Skill Quiz B: Arithmetic

Question 6.

The answer is (A).

Explanation for Question 6:

There are $4 \times 60 = 240$ minutes in 4 hours. You then divide by 8 to get the number of octaminutes. $240 \div 8 = 30$. The answer is (A).

Question 7.

The answer is (C).

Explanation for Question 7:

Hopefully you found out you divide to get the answer. Change \$3.00 to cents (300 cents) and then divide by p cents to get $300/p$. The answer is (C).

Question 8.

The answer is (B).

Explanation for Question 8:

Divide 3,500 by 300. $3,500 \div 300 = 11.666\ldots$ Only whole gallons can be used so you have to use the next higher integer, 12, for the answer. The answer is (B).

Question 9.

The answer is (E).

Explanation for Question 9:

The closest numbers are 3.00 and 20. The answer is (E).

Question 10.

The answer is 10.

Explanation for Question 10:

Start with the units digits. $3 + 5 + \underline{4} = 12$. To get a 2, C must equal 4.
Carry the 1 to the tens digits. Next $1 + B + 6 + 9 = 7$. B must be a 1.
Carry the 1 to the hundreds digits. Next $1 + 9 + A + 3 = B8$.
A must be a 5 and B is still a 1. So $B = 1$, $C = 4$, and $A = 5$ their sum is 10.
The answer is 10.



Question 1

Which of the following is equal to 4?

- (A) $16 / [8 - (6 \times 2)]$
- (B) $[16 / (8 - 6)] \times 2$
- (C) $[(16 / 8) - 6] \times 2$
- (D) $16 / [(8 - 6) \times 2]$
- (E) $(16 / 2) - (6 \times 2)$

Question 2

If lace costs \$0.42 per yard, how many yards of lace can be bought for \$7.35?

- (A) $5 \frac{7}{10}$
- (B) 6
- (C) $12 \frac{1}{2}$
- (D) 17
- (E) $17 \frac{1}{2}$

Question 3

In Italy, when 1 dollar was approximately equal to 1,800 lire, a certain shirt cost 46,000 lire. Of the following, which is the best approximation of the cost of the shirt, in dollars?

- (A) 10
- (B) 15
- (C) 20
- (D) 25
- (E) 30

Question 4

Sally buys paper and a pen for \$2.09. If the pen costs \$0.31 more than the paper, how much does the paper cost?

- (A) 0.20
- (B) 0.31
- (C) 0.89
- (D) 1.20
- (E) 2.09

Question 5

On planet Xydo, if each year has 6 months and each month has 21 days, how many years will have passed after 882 days?

- (A) 1
- (B) 6
- (C) 7
- (D) 42
- (E) 147

Question 6

1st Row	0	0	0	1	0	0	0
2nd Row	0	0	1	1	1	0	0
3rd Row	0	1	2	3	2	1	0
4th Row	0	3	6	7	6	3	0
5th Row	0	—	—	—	—	—	0

Beginning with the second row, each number in a row shown is the sum of the three numbers nearest to it in the row immediately above. If a fifth row is added in this fashion, what will be the sum of all the numbers in the fifth row?

- (A) 25
- (B) 36
- (C) 49
- (D) 64
- (E) 69

Question 7

In the ABC Preschool, children must be no more than 4 years old on the start date of August 1. What is the oldest, in months, that a child can be on April 1 and still start school on the following August 1?

- (A) 42
- (B) 43
- (C) 44
- (D) 45
- (E) 46

Question 8

How many three-digit numbers have the hundreds digit equal to 4 and the units digit equal to 5?

- (A) 10
- (B) 19
- (C) 20
- (D) 190
- (E) 200

Question 9

Speed (in miles per hour)	Thinking Distance (in feet)	Braking Distance (in feet)
20	20	20
30	30	45
40	40	80
50	50	125
60	60	180

The table above can be used to calculate the distance required to stop a car traveling at a given speed by adding the thinking distance and the braking distance. How many more feet does it take to stop a car traveling at 60 miles per hour than at 30 miles per hour?

- | | |
|---------|---------|
| (A) 30 | (D) 165 |
| (B) 75 | (E) 240 |
| (C) 135 | |

Grid-in your answer here:

Question 10

$$\begin{array}{r} 24P \\ +P3 \\ \hline 26R \end{array}$$

In the above problem P and R represent digits in the correctly worked addition problem. What does R represent?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9



Hint for Question 1:

Don't forget to use order of operations. If you're using a calculator be sure it follows order of operations.

Hint for Question 2:

What operation do you use?

Hint for Question 3:

What operation do you use?

Hint for Question 4:

You could use algebra to get an equation to solve or just substitute the answer choices until you get the correct answer.

Hint for Question 5:

How many days are in a year on Xydo?

Hint for Question 6:

You're looking for patterns. What are the numbers on the 5th row?

Hint for Question 7:

4 years is how many months?

Hint for Question 8:

One way is to quickly write them all down.

Hint for Question 9:

Be sure you read the question carefully. How many feet does it take to stop at 60 miles per hour? How many feet does it take to stop at 30 miles per hour?

Hint for Question 10:

Substitute numbers for P and R until you find ones that work. Which variable should you substitute for first?



Question 1.

The answer is (D).

Explanation for Question 1:

Remember to do the operations inside the brackets and parentheses first.

$$16 / [(8 - 6) \times 2] = 16 / [2 \times 2] = 16 / 4 = 4$$

The answer is (D).

Question 2.

The answer is (E).

Explanation for Question 2:

You know the price for 1 yard and the total price. You divide to get the answer. $7.35 \div 0.42 = 17.5$. The answer is (E).

Question 3.

The answer is (D).

Explanation for Question 3:

This is similar to question number 2. You know the worth of 1 dollar and the total worth. You divide to get the answer. $46,000 \div 1,800 = 25$.

The answer is (D).

Question 4.

The answer is (C).

Explanation for Question 4:

Using algebra, let x be the cost of the paper. The pen then costs $x + 0.31$.

Then solve the equation $x + x + 0.31 = 2.09$.

The answer is (C).

OR

Because the answer choices are in order from least to

greatest start with choice (C) and substitute into the problem. Paper costs 0.89.

The pen costs $0.89 + 0.31 = 1.20$. The pen + Paper = 2.09. We're lucky! We got it on the first try. If this answer was too big we'd next try (A) or (B). If this answer was too small we'd next try (D) or (E).

Question 5.

The answer is (C).

Explanation for Question 5:

The number of days in one year on Xydo is $6 \times 21 = 126$. To find the number of years in 882 days you have to divide 882 by 126. $882 \div 126 = 7$. The answer is (C).

Question 6.

The answer is (E).

Explanation for Question 6:

To find the 5th row

$$\text{Empty space 1} = 0 + 3 + 6 = 9$$

$$\text{Empty space 2} = 3 + 6 + 7 = 16$$

$$\text{Empty space 3} = 6 + 7 + 6 = 19$$

$$\text{Empty space 4} = 7 + 6 + 3 = 16$$

$$\text{Empty space 5} = 6 + 3 + 0 = 9$$

To finish the problem you have to add $9 + 16 + 19 + 16 + 9 = 69$. The answer is (E).

Question 7.

The answer is (C).

Explanation for Question 7:

The child must be 48 months old on August 1. April 1st is 4 months before August 1st. $48 - 4 = 44$. The answer is (C).

Question 8.

The answer is (A).

Explanation for Question 8:

The numbers are 405, 415, 425, 435, ..., 495. If you count them you should get 10. Another way is to notice the tens digits of 0 through 9. There are 10 of these digits so you still should get 10. If you understood the second way this is a quick question to answer. The answer is (A).

Question 9.

The answer is (D).

Explanation for Question 9:

At 60 miles per hour it takes $60 + 180 = 240$ feet to stop. At 30 miles per hour it takes $30 + 45 = 75$ feet to stop. To answer the question you subtract. $240 - 75 = 165$. The answer is (D).

Question 10.

The answer is 5.

Explanation for Question 10:

First substitute for the P in the tens place. It can only be a 2. Then the value of R must be a 5. The answer is 5.



Grid-in your answer here:

Question 1

Of a set of 48 pencils, $\frac{1}{3}$ of them are red. Exactly 10 of the red pencils have erasers. The rest of the red pencils do not have erasers. How many of the red pencils do not have erasers?

.	/	/	.	.
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Question 2

A number is given whose tens digit is t and whose units digit is u . If the digit 5 is placed after these two digits, the value of the new number is

- (A) $t + 5$
(B) $10t + u + 5$
(C) $10t + 10u + 5$
(D) $100t + 10u + 5$
(E) $100t + u + 50$

Question 3

If wood costs \$0.64 per square foot, how many square feet of wood can be bought for \$16.00?

- (A) 4
(B) 8
(C) $20\frac{2}{3}$
(D) 25
(E) $25\frac{1}{2}$

Question 4

$$\begin{array}{r} 37 \\ \times 2A \\ \hline 1CB \\ \hline 74 \\ \hline 92B \end{array}$$

If A, B, and C each represent a digit in the correctly worked problem above, what digit does A represent?

- (A) 3
- (B) 4
- (C) 5
- (D) 6
- (E) 7

Question 5

If $3x$ is an odd integer, then x could be any of the following except

- (A) -3
- (B) 4
- (C) 5
- (D) 7
- (E) 9

Question 6

In a two-digit number r is the tens digit and s is the units digit, if 10 is added to the number the resulting number could be expressed as

- (A) $10r + s$
- (B) $10(r + 1) + s$
- (C) $10r + 10s$
- (D) $10r + s + 1$
- (E) $11r + s$

Studyguide for the SAT Skill Quiz D: Arithmetic

Question 7

Through how many degrees does a minute hand of a clock turn from 6:40 a.m. to 6:55 a.m.?

- (A) 20
- (B) 90
- (C) 120
- (D) 180
- (E) 240

Grid-in your answer here:

Question 8

April packed 360 bottles of purified water in cartons of 12 bottles each and Bruce packed 360 bottles of purified water in cartons of 30 bottles each. How many more cartons did April use than Bruce used?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Grid-in your answer here:

Question 9

How many bottles, each holding 16 fluid ounces, are needed to hold 10 quarts of orange juice? (1 quart = 32 fluid ounces)

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Grid-in your answer here:

Question 10

Typically, five oranges of a certain variety weigh 1 pound. Twenty pounds of these oranges cost \$18.00. At this rate, what is the cost, in dollars, of two dozen oranges?
(This is a grid-in problem. Disregard the dollar sign when you grid your answer. If, for example, your answer is \$1.34, grid 1.34.)

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Studyguide for the SAT Skill Quiz D: Arithmetic



Hint for Question 1:

Be sure to answer the question asked. First find out how many pencils are red.

Hint for Question 2:

Look at the answers. With 5 placed on the end of the number what would the tens digit be? The units digit? What digit would it be?

Hint for Question 3:

Think about what operation you should use.

Hint for Question 4:

Substitute numbers for A, B, and C until you find ones that work. Think about which variable you should substitute for first.

Hint for Question 5:

What is the operation between the 3 and the x? Substitute the answer choices to find the one that gives a product that is not odd.

Hint for Question 6:

Express the number as $10r + s$ and then add 10. Which of the choices is equivalent to your solution?

Hint for Question 7:

One way to solve this is to first figure how many degrees are in one minute.

Hint for Question 8:

Be sure to answer the question asked. Think about what operations you should use.

Hint for Question 9:

Be sure the units are the same. Think about what operations you should use.

Hint for Question 10:

This problem requires multiple steps to solve. One way is to find the cost per orange. Think what steps you would need to take to find the cost per orange.



Question 1.

The answer is 6.

Explanation for Question 1:

The number of red pencils is $48 \times \frac{1}{3} = 16$. Subtract the number with erasers
 $16 - 10 = 6$. The answer is 6.

Question 2.

The answer is (D).

Explanation for Question 2:

The t is now the hundreds digit, u is the tens digit, and 5 is the units digit. The value is $100t + 10u + 5$. The answer is (D).

Question 3.

The answer is (D).

Explanation for Question 3:

You should divide \$16.00 by \$0.64. $16.00 \div .64 = 25$. The answer is (D).

Question 4.

The answer is (C).

Explanation for Question 4:

C is the variable you should find first. It has to be the number 8. Now you can find A. $A \times 37$ has to equal $18B$. The only number for A that will give you the 1 and 8 is 5. So A must be 5. B must be 5 also but you weren't asked to find B. The answer is (C).

Question 5.

The answer is (B).

Explanation for Question 5:

The product of 3 and x is odd for all choices except 4. The answer is (B).

Question 6.

The answer is (B).

Explanation for Question 6:

The resulting number is $10r + s + 10$. The choice that is equivalent to this number is (B). (Don't forget to distribute the 10 for answer (B).)
The answer is (B).

Question 7.

The answer is (B).

Explanation for Question 7:

The minute hand moves 6 degrees in 1 minute.
 $360 \text{ degrees} \div 60 \text{ minutes} = 6 \text{ degrees per minute.}$
There are 15 minutes from 6:40 a.m. to 6:55 a.m.
So the minute hand moves $6 \times 15 = 90$ degrees.
The answer is (B).

Question 8.

The answer is 18.

Explanation for Question 8:

April used $360 \div 12 = 30$ cartons. Bruce used $360 \div 30 = 12$ cartons. April used $30 - 12 = 18$ more cartons than Bruce. The answer is 18.

Question 9.

The answer is 20.

Explanation for Question 9:

You need to change 10 quarts to ounces. $10 \text{ quarts} = 10 \times 32 \text{ ounces} = 320 \text{ ounces.}$
Now you can divide 320 ounces by 16 ounces. $320 \div 16 = 20.$ The answer is 20.

Question 10.

The answer is 4.32.

Explanation for Question 10:

The cost per orange can be found by doing the following:
20 pounds of oranges cost \$18.00
From this information 1 pound of oranges costs \$0.90. ($18.00 \div 20 = .90$)
There are 5 oranges in 1 pound so by substituting this information, you now know 5 oranges cost \$0.90.
From this information, you find that 1 orange costs \$0.18. ($.90 \div 5 = .18$)
Two dozen oranges must then cost $24 \times \$0.18 = \$4.32.$
Your directions say to disregard the dollar sign in the answer.
The answer is 4.32.



DIVISIBILITY

The factors of an integer X are the integers that X can be divided by and have no remainder. X is said to be divisible by its factors.

Example:

10 is divisible by 5 and 2. $10 \div 2 = 5$ with no remainder.

$10 \div 5 = 2$ with no remainder. 10 is not divisible by 3.

$10 \div 3 = 3$ with a remainder of 1.

Divisibility Rules

It will be helpful to remember the following rules for divisibility:

1. A number is divisible by 2 if it ends in an even number.
2. A number is divisible by 3 if the sum of its digits is divisible by 3.
(The number 123 is divisible by 3, because $1 + 2 + 3 = 6$, which is divisible by 3.)
3. A number is divisible by 4 if its last two digits form a number divisible by 4. The number 324 is divisible by 4, because its last two digits form a number, 24, that is divisible by 4.
4. A number is divisible by 5 if it ends with a 0 or 5.
5. A number is divisible by 9 if the sum of its digits is divisible by 9.
The number 369 is divisible by 9, because $3 + 6 + 9 = 18$, which is divisible by 9.
6. A number is divisible by 10 if it ends with a 0.

Example 1:

What integer greater than 1 are both 200 and 150 divisible by?

- (A) 3
- (B) 7
- (C) 30
- (D) 50
- (E) 300

Hint:

If an integer N is divisible by M , then N is also divisible by any factors of M . For example, 100 is divisible by 20, so 100 is also divisible by any factors of 20 (such as 4 and 5). Find all the factors of 200 and 150.

Solution:

The positive integral factors of 200 are 1, 2, 4, 5, 10, 20, 40, 50, 100, and 200. The positive integral factors of 150 are 1, 2, 3, 5, 10, 15, 30, 50, 75, and 150. The answer choice that both are divisible by is 50.

Example 2:

If a large case of pencils can be equally divided among 20 students, among what other number of students can the same case of pencils always be equally divided?

- (A) 3
- (B) 5
- (C) 7
- (D) 9
- (E) 11

Hint:

If an integer N is divisible by M , then N is also divisible by any factors of M .

Solution:

Because the total number of pencils is divisible by 20, it is always divisible by any factors of 20, such as 5. The answer is (B).

Example 3:

When x is divided by 11 its remainder is 4. What is the remainder when $5x$ is divided by 11?

- (A) 2
- (B) 3
- (C) 4
- (D) 7
- (E) 9

Hint:

One way to solve these types of problems is to pick a number for x that fits the description.

Solution:

One value you could pick for x is 15 because 15 divided by 11 has a remainder of 4. Then $5x$ would be 75 and 75 divided by 11 has a remainder of 9. The answer is (E).



Question 1

There are fewer than 40 students enrolled in a certain class. If, at a certain time, $\frac{1}{7}$ of the students are members of the band and another $\frac{1}{5}$ are members of sports teams, what is the total class enrollment?

- (A) 10
- (B) 14
- (C) 25
- (D) 28
- (E) 35

Question 2

How many different positive integral divisors does 24 have including 1 and 24?

- (A) 8
- (B) 7
- (C) 6
- (D) 4
- (E) 2

Question 3

A number is divisible by 3 if the sum of its digits is divisible by 3. Which of the following numbers is divisible by 15?

- (A) 21,115
- (B) 24,048
- (C) 32,715
- (D) 33,333
- (E) 72,365

Studyguide for the SAT Skill Quiz A: Divisibility

Question 4

If $r \times s \times t = 360$ where r , s , and t are integers with $r > s > t > 1$, what is the greatest possible value of r ?

- (A) 180
- (B) 60
- (C) 36
- (D) 24
- (E) 12

Question 5

If x is divided by 8, the remainder is 7. What is the remainder if $4x$ is divided by 8?

- (A) 5
- (B) 4
- (C) 3
- (D) 2
- (E) 1

Question 6

Eggs are packed exactly 1 dozen to a carton, and extras are left for the next packing. If 206 eggs are ready to be packed and if there are plenty of cartons, how many eggs will be left for the next packing?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

Question 7

If R is divisible by 2 and S is divisible by 3, which of the following must RS be divisible by?

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 9

Question 8

If R is replaced by one of the digits listed below, the given number will be divisible by 2, 3, 4, 5, and 9. What is the value of R?

92,3R0

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

Question 9

If $1,000 \geq r \geq 400$ and r is a multiple of 12, 15, 16, and 18, then $r =$

- (A) 480
- (B) 600
- (C) 720
- (D) 840
- (E) 960

Question 10

When 35 is divided by 6, the remainder is the same as when 82 is divided by which of the following numbers?

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8

Studyguide for the SAT Skill Quiz A: Divisibility



Hint for Question 1:

What number less than 40 is divisible by both 5 and 7?

Hint for Question 2:

Find all of the divisors (factors) of 24. Remember the question asks for how many divisors.

Hint for Question 3:

To be divisible by 15 the number must be divisible by 3 and what other number?

Hint for Question 4:

What are the factors of 360?

Hint for Question 5:

Find a number for x that fits the problem.

Hint for Question 6:

How many eggs are in a dozen? What operation do you use?

Hint for Question 7:

RS indicates a product. What do you do with the divisors?

Hint for Question 8:

Go over your divisibility rules. What number must R be to make the rules for 2, 3, 4, 5, and 9 true?

Hint for Question 9:

How do you check to see if a number is a multiple of another?

Hint for Question 10:

What is the remainder when 35 is divided by 6?



Question 1.

The answer is (E).

Explanation for Question 1:

The answer choice is 35. It is divisible by both 5 and 7. The answer is (E).

Question 2.

The answer is (A).

Explanation for Question 2:

The divisors are 1, 2, 3, 4, 6, 8, 12, and 24. There are 8 of them. The answer is (A).

Note: The question could have asked for the sum of the positive integral divisors.

Question 3.

The answer is (C).

Explanation for Question 3:

The number must be divisible by 3 and 5. The number must then end in a 5 or 0. You can exclude answers (B) and (D). Quickly add the digits in answer (A) — you get 10; add the digits in answer (C) — you get 18. 18 is divisible by 3, so (C) must be the answer. The answer is (C). (If you think like this you can do the problem faster without a calculator.)

Question 4.

The answer is (B).

Explanation for Question 4:

Divide 360 by each of the answer choices and figure out what s and t would be.

For choice (A): $\frac{360}{180} = 2$. This means $s = 2$ and $t = 1$.

Because neither t or s can equal 1, this cannot be a choice. For choice (B):

$\frac{360}{60} = 6$. This means s would = 3 and t would = 2. These are the

smallest values for s and t, so r must be the largest. The answer is (B).

Question 5.

The answer is (B).

Explanation for Question 5:

One choice for x would be 15. $4x$ is equal to 60 and $\frac{60}{8}$ has a remainder of 4. The answer is (B).

Question 6.

The answer is (B).

Explanation for Question 6:

There are 12 eggs in a dozen and you divide to find out how many cartons to use. The remainder will be the number of eggs left over for the next packing. (Note: If you use a calculator you'll get a decimal instead of a remainder. Long division is needed to answer this problem or know how to get the decimal part to an integer.)

$$\frac{206}{12} = 17 \text{ with a remainder of } 2. \text{ The answer is (B).}$$

Question 7.

The answer is (C).

Explanation for Question 7:

RS is a product so a divisor of RS must be the product of the divisor of R and the divisor of S. The product of their divisors is $2 \times 3 = 6$. The answer is (C).

Question 8.

The answer is (D).

Explanation for Question 8:

Because the number ends in 0 it is automatically divisible by 2 and 5. To be divisible by 4 the last two digits must be divisible by 4. That helps us eliminate choices (A), (C), and (E). To be divisible by 3 and 9 the sum of the digits must be divisible by 9. $9 + 2 + 3 + 0 = 14$. If $R = 4$ then $14 + 4 = 18$ is divisible by 9. The answer is (D).

Question 9.

The answer is (C).

Explanation for Question 9:

One way to solve the problem is to use your calculator and divide each answer choice by 12, 15, 16, and 18. Another, faster way, is to see if 2, 3, 4, 5, and 9 are divisors of the answer choices. (Can you see why only these need to be picked?) The numbers 0, 2, and 5 are divisors for all choices. The last two digits for all answer choices are divisible by 4, so 4 is a divisor of all choices. The only thing left is to see which choice has a divisor of 9. You don't have to try 3 because 3 is a divisor of 9. Add up the digits and see which choice is divisible by 9. The only one divisible by 9 is 720. $7 + 2 + 0 = 9$. The answer is (C).

Question 10.

The answer is (D).

Explanation for Question 10:

Try to divide in your head quickly. You shouldn't need a calculator. The remainder of $\frac{35}{6}$ is 5. Try each answer choice to see which one yields a remainder of 5.

The remainder when 82 is divided by 7 is 5. The answer is (D). Using a calculator can make this problem harder, not easier.



Question 1

There are fewer than 40 students enrolled in a certain class. If, at a certain time, $\frac{1}{11}$ of the students are members of the soccer team and another $\frac{1}{3}$ are members the chess team, what is the total class enrollment?

- (A) 11
- (B) 14
- (C) 21
- (D) 22
- (E) 33

Question 2

A number is divisible by 11 if by alternately subtracting and adding its digits you get a number divisible by 11. Example: 924 is divisible by 11 because $9 - 2 + 4 = 11$ and 11 is divisible by 11. (Don't forget 0 is divisible by 11.) Which of the following numbers is divisible by 11?

- (A) 721
- (B) 3,244
- (C) 8,731
- (D) 11,111
- (E) 85,723

Question 3

If a is divided by 5, the remainder is 2. What is the remainder if $3a$ is divided by 5?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

Question 4

Sodas are packed exactly 24 to a case, and extras are left for the next packing. If 206 sodas are ready to be packed and if there are plenty of cases, how many sodas will be left for the next packing?

- (A) 22
- (B) 20
- (C) 14
- (D) 12
- (E) 4

Question 5

R is divisible by 3 and S is divisible by 5. Which of the following could be divisible by RS?

- (A) 185
- (B) 310
- (C) 711
- (D) 1,455
- (E) 3,565

Question 6

When a positive integer x is divided by 7, the remainder is 2. Which of the following expressions will yield a remainder of 4 when x is divided by 7?

- (A) $x + 1$
- (B) $x + 2$
- (C) $x + 3$
- (D) $x + 4$
- (E) $x + 5$

Question 7

If a , b , and c are integers greater than 1, where $ab = 10$ and $ac = 35$, which of the following must be true?

- (A) $c > a > b$
- (B) $b > c > a$
- (C) $b > a > c$
- (D) $a > b > c$
- (E) $c > b > a$

Studyguide for the SAT Skill Quiz B: Divisibility

Question 8

The product of two integers is between 206 and 211. Which of the following **cannot** be one of the integers?

- (A) 5
- (B) 7
- (C) 9
- (D) 17
- (E) 23

Grid-in your answer here:

Question 9

The number 76 is divisible by x where $1 < x < 76$.
What is one possible value of x ?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Grid-in your answer here:

Question 10

The product of 3 and x is divisible by 6.
If the product is between 20 and 29, what is one possible value of x ?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Studyguide for the SAT Skill Quiz B: Divisibility



Hint for Question 1:

What number less than 40 is divisible by both 3 and 11?

Hint for Question 2:

Alternate subtracting and adding the digits. Example: $7 - 2 + 1 = 6$ because 6 is not divisible by 11, 721 is not divisible by 11.

Hint for Question 3:

Pick a number that works for a.

Hint for Question 4:

What operation do you use? Look for the remainder!

Hint for Question 5:

The answer choice has to be divisible by what two numbers?

Hint for Question 6:

Pick a number for x that fits the problem and figure how much greater the number must be to get a remainder of 4 instead of 2.

Hint for Question 7:

What are number choices for a and b? What are number choices for a and c?

Hint for Question 8:

How could you use your calculator for help?

Hint for Question 9:

This problem is checking to see if you know what "divisible" means. The number 76 is divisible by what numbers?

Hint for Question 10:

What numbers do you multiply by 3 to get an answer between 20 and 29? Which of these fits the condition that 3 times it is divisible by 6?



Question 1.

The answer is (E).

Explanation for Question 1:

The number that is divisible by both 3 and 11 is 33. The answer is (E).

Question 2.

The answer is (E).

Explanation for Question 2:

$8 - 5 + 7 - 2 + 3 = 11$. This is the only one divisible by 11. The answer is (E).

Question 3.

The answer is (A).

Explanation for Question 3:

Let $a = 7$ because 7 divided by 5 has a remainder of 2. $3a$ is equal to 21, and when it is divided by 5 the remainder is 1. The answer is (A). You could also do algebra. Let $a = 5n + 2$ (n is the number of times 5 would divide into a and 2 is the remainder), then $3a = 15n + 6$. When $15n + 6$ is divided by 5, you get a remainder only from the 6. The remainder will always be 1.

Question 4.

The answer is (C).

Explanation for Question 4:

You must divide 206 by 24. The answer to the problem is the remainder. The remainder is 14. The answer is (C).

Question 5.

The answer is (D).

Explanation for Question 5:

The answer has to be divisible by 3 and 5. So the answer has to end in a 5 or 0 and the sum of the digits must be divisible by 3. (A), (C), (D), and (E) are possible choices, but (D) is the only answer that has the sum of the digits divisible by 3. The answer is (D).

Studyguide for the SAT Skill Quiz B: Divisibility

Question 6.

The answer is (B).

Explanation for Question 6:

A number to pick for x could be 9 because 9 divided by 7 has a remainder of 2. The number with a remainder of 4 when divided by 7 would then be 11 and $11 = 9 + 2$. So the answer must be two greater than x . Or...a remainder of 4 is two greater than a remainder of 2. The answer must be two greater than x . The answer is (B).

Question 7.

The answer is (A).

Explanation for Question 7:

Answer choices for a and b can be only 2 and 5. Answer choices for a and c can only be 5 and 7. From the first choice a is either 2 or 5 and from the second choice a is either 5 or 7. From these choices we deduct that a must be 5. Because a is 5, then b must be the number 2 and c must be the number 7. The greatest number is then c , and a is greater than b . The answer is (A).

Question 8.

The answer is (D).

Explanation for Question 8:

You can divide 206 and 211 by each answer choice. Record your answer for both 206 and 211. If there is an integer between these two answers then it is the integer that can be multiplied with the answer choice to get a number between 206 and 211. Remember you're looking for a number that doesn't work.

Choice	206	211	Integer between	Check
(A)	$\frac{206}{5} = 41.2$	$\frac{211}{5} = 42.2$	42	$5 \times 42 = 210$
(B)	$\frac{206}{7} = 29.4$	$\frac{211}{7} = 30.1$	30	$7 \times 30 = 210$
(C)	$\frac{206}{9} = 22.9$	$\frac{211}{9} = 23.4$	23	$9 \times 23 = 207$
(D)	$\frac{206}{17} = 12.1$	$\frac{211}{17} = 12.4$	none	

The answer is (D).

Studyguide for the SAT Skill Quiz B: Divisibility

Question 9.

The answer is 2, 4, 19, or 38.

Explanation for Question 9:

The number 76 is divisible by 2, 4, 19, and 38. Any of these four numbers will answer the problem. The answer is 2, 4, 19, or 38.

Question 10.

The answer is 8.

Explanation for Question 10:

The numbers for x are 7, 8, and 9. ($3 \times 7 = 21$, $3 \times 8 = 24$, $3 \times 9 = 27$)

The only product that is divisible by 6 is 24. So x must be 8.

The answer is 8.



RATIOS

There will be a few questions on the SAT about ratios.

A ratio tells you the relative number of parts.

If the ratio of two quantities is $a:b$, then a and b represent the relative number of parts of the two quantities.

The ratio tells you that there are

" a parts of the first quantity," and

" b parts of the second quantity."

Ratios only tell you the relative size - not the actual quantities.

For example, if you say you have twice as many apples as oranges, that doesn't tell us how many of each you have, just the relative amounts of each.

(It doesn't mean you only have 2 apples and 1 orange.

You could have 8 apples and 4 oranges.)

Example 1:

If the ratio of boys to girls in a classroom is $2:1$, how many of each are in a class of 15 students.

Solution:

There are 3 parts total (2 parts boys, 1 part girls).

If there are 15 students total, there are

$$\frac{15 \text{ students}}{3 \text{ parts}} = 5 \text{ students per "part."}$$

Therefore,

$$(2 \text{ parts boys}) \times (5 \text{ students per part}) = 10 \text{ boys}$$

$$(1 \text{ part girls}) \times (5 \text{ students per part}) = 5 \text{ girls}$$

Example 2:

The ratio of flour to corn meal in a recipe is 3:2.

If the total amount of corn meal plus flour is 10 cups,
how many of the cups are flour?

Solution:

The total number of parts is $3 + 2 = 5$.

Divide the 10 cups by 5 parts. There are

$$\frac{10 \text{ cups}}{5 \text{ parts}} = 2 \text{ cups per part.}$$

The number of cups of flour is $(2 \text{ cups per part}) \times (3 \text{ parts flour}) = 6 \text{ cups of flour.}$

Converting Ratios to Fractions

A ratio also can be expressed as a fraction. For example, if the ratio of boys to girls in a classroom is 3:1, there are 4 parts total, so $\frac{3}{4}$ of the class are boys and $\frac{1}{4}$ of the class are girls.

If there are 16 students, there are $\frac{3}{4} \times 16 = 12$ boys and $\frac{1}{4} \times 16 = 4$ girls.

Example 3:

A basket contains 10 balls, some white and some red.

Which of the following is NOT a possible ratio of white to red balls?

(A) 1:1 (B) 2:1 (C) 4:1 (D) 1:4

(E) All of the above are possible ratios.

Solution:

In this problem you must check each ratio to see whether there is some combination of white and red balls that has the specified ratio and adds up to 10.

The answer choice that is not a possible ratio is correct.

Choice (A) 1:1 Total of 2 parts. $10/2 = 5$. (5 white balls): (5 red balls)

Choice (B) 2:1 Total of 3 parts. $10/3$ is not an integer. This ratio cannot work.

(In reality you would stop here. Your answer is (B), but let's check the other ratios.)

Choice (C) 4:1 Total of 5 parts. $10/5 = 2$. (8 white balls): (2 red balls)

Choice (D) 1:4 Total of 5 parts. $10/5 = 2$. (2 white balls): (8 red balls)

Studyguide for the SAT Skill Review: Ratios

Sometimes you may be asked to compare ratios of lengths of sides of similar figures to areas of the figures or even volumes of similar solids. These questions may seem difficult but are really easy once you understand the concept.

Example 4:

If the sides of two squares are in the ratio of 2:7,
what is the ratio of their areas?

Solution:

First: Squares are always "similar" figures. Remember similar means corresponding angles are equal and corresponding sides are in the same ratio.

Second: To find the area of a square you square a side. If the squares are in a ratio of 2:7 that means a side of the smaller square could be $2x$ and the side of the larger square could be $7x$. $2x:7x$ reduces to the given ratio of 2:7.

The area of the smaller square is then $(2x)^2 = 4x^2$ and

the area of the larger square is $(7x)^2 = 49x^2$. The ratio

of their areas is $4x^2 : 49x^2 = 4:49$. The x^2 's cancel.

The point is you need only square the ratio of the lengths of the sides to find the ratio of the areas.

Squares were used in this example but this concept is true no matter what the shape of the similar figures.

Example 5:

If two similar triangles have bases in the ratio of 5:4,
what is the ratio of their areas?

Solution:

The ratio of their areas is the square of the ratio of their sides (bases). The ratio of their areas is 25:16.



Question 1

When the package of grapes was opened, there were 36 grapes total. The first child ate $\frac{1}{3}$ of the grapes and the second child ate $\frac{1}{3}$ of the remainder. How many grapes remain?

- (A) 12
- (B) 16
- (C) 20
- (D) 24
- (E) 32

Question 2

The ratio of two numbers is 3:7. If the larger number is 35, what is the smaller number?

- (A) 3
- (B) 4
- (C) 7
- (D) 10
- (E) 15

Question 3

The ratio of girls to boys in the class is 2:1.

If there are 30 students in this class, how many are girls?

- (A) 2
- (B) 10
- (C) 18
- (D) 20
- (E) 32

Question 4

If the ratio of yellow jellybeans to red jellybeans is 4:3 and there are 21 jellybeans in the jar, how many are yellow?

- (A) 7
- (B) 9
- (C) 12
- (D) 14
- (E) 18

Question 5

The ratio of white socks in a drawer to blue socks is 3:4.

If there are 6 white socks in the drawer, how many blue socks are there?

- (A) 3
- (B) 4
- (C) 6
- (D) 7
- (E) 8

Question 6

During the month of May the ratio of grapes to bananas to apples sold in a grocery store was 2:3:5. If the store sold 30 apples in May, how many grapes, bananas, and apples did the store sell in all?

- (A) 30
- (B) 35
- (C) 45
- (D) 50
- (E) 60

Question 7

If the ratio of pecans to cherries in a salad is 3 to 5, which of the following cannot be the total number of pecans and cherries in the salad?

- (A) 32
- (B) 36
- (C) 40
- (D) 48
- (E) 56

Question 8

If the ratio of cats to dogs in a certain pound is 5:3, and there are 65 cats, how many dogs are in the pound?

- (A) 13
- (B) 18
- (C) 26
- (D) 36
- (E) 39

Question 9

After spending $\frac{5}{12}$ of her monthly income, Carole has \$140 left.

What is her monthly income?

- (A) 200
- (B) 240
- (C) 300
- (D) 420
- (E) 583

Question 10

In a schoolwide election, votes were cast for Mrs. Mitchell, Mr. Madison, and Mrs. Brinson in the ratio of 4:3:2. If these were the only candidates and 1,800 students voted for only one candidate each, how many voted for Mr. Madison?

- (A) 200
- (B) 300
- (C) 400
- (D) 600
- (E) 900

Studyguide for the SAT Skill Quiz A: Ratios



Hint for Question 1:

How many grapes were left after the first child ate $\frac{1}{3}$?

Hint for Question 2:

You can solve this by $\frac{3}{7} \times 35$.

Hint for Question 3:

Use a ratio table if you have problems.

Hint for Question 4:

Try a ratio table.

Hint for Question 5:

Make a ratio table.

Hint for Question 6:

Try a ratio table.
Remember you are after the total.

Hint for Question 7:

The answer has to do with the total number of parts.

Hint for Question 8:

Use a ratio table.

Hint for Question 9:

If Carole spent $\frac{5}{12}$ of her monthly income, she has $\frac{7}{12}$ left.

Therefore, $\frac{7}{12}$ equals \$140.

Hint for Question 10:

$$4 + 3 + 2 = 9$$



Answers!



Question 1.

The answer is (B).

Explanation for Question 1:

There were 24 grapes after the first $\frac{1}{3}$ were eaten.

$\frac{1}{3}$ of 24 is 8. So there were $\frac{2}{3}$ of 24, or 16, left.

The answer is (B).

Question 2.

The answer is (E).

Explanation for Question 2:

$$\frac{3}{7} \times 35 = 15.$$

15:35 is the same ratio as 3:7.

The answer is (E).

Studyguide for the SAT Skill Quiz A: Ratios

Question 3.

The answer is (D).

Explanation for Question 3:

Your ratio table should look like this:

Step 1.

Girls	Boys	Students
2	1	—
—	—	—
—	—	30

Step 2.

Girls	Boys	Students
2	1	3
10	10	10
—	—	30

Step 3.

Girls	Boys	Students
2	1	3
10	10	10
20	10	30

There are 20 girls.

The answer is (D).

Studyguide for the SAT Skill Quiz A: Ratios

Question 4.

The answer is (C).

Explanation for Question 4:

Your ratio table should look like this

Step 1.

Yellow	Red	Total
4	3	—
—	—	—
—	—	21

Step 2.

Yellow	Red	Total
4	3	7
3	3	3
—	—	21

Step 3.

Yellow	Red	Total
4	3	7
3	3	3
12	9	21

There are 12 yellow jelly beans.

The answer is (C).

Studyguide for the SAT Skill Quiz A: Ratios

Question 5.

The answer is (E).

Explanation for Question 5:

Your ratio table should look like this

Step 1.

White	Blue	Total
3	4	—
—	—	—
6	—	—

Step 2.

White	Blue	Total
3	4	7
2	2	2
6	—	—

Step 3.

White	Blue	Total
3	4	7
2	2	2
6	8	14

There are 8 blue socks. The answer is (E).

Studyguide for the SAT Skill Quiz A: Ratios

Question 6.

The answer is (E).

Explanation for Question 6:

Step 1.

Grapes	Bananas	Apples	Total
2	3	5	—
—	—	—	—
—	—	30	—

Step 2.

Grapes	Bananas	Apples	Total
2	3	5	10
6	6	6	6
—	—	30	—

Step 3.

Grapes	Bananas	Apples	Total
2	3	5	10
6	6	6	6
12	18	30	60

The answer is (E).

Question 7.

The answer is (B).

Explanation for Question 7:

Thirty-six is the only answer that is not a multiple of the total number of parts (8).

The answer is (B).

Question 8.

The answer is (E).

Explanation for Question 8:

Set up a ratio table.

The middle row will be 13.

$3 \times 13 = 39$. The answer is (E).

Question 9.

The answer is (B).

Explanation for Question 9:

Use the hint to solve the problem.

Let x be Carole's monthly income. The equation that represents the given is

$$(7/12)x = 140$$

$x = (12/7)140 = 240$. The answer is (B).

Question 10.

The answer is (D).

Explanation for Question 10:

The sum of the 3 parts of the ratio 4:3:2 is 9.

The total number of votes, 1,800, divided by 9 gives a factor of 200.

There were 800 votes for Mrs. Mitchell, 600 votes for Mr. Madison and 400 votes for Mrs. Brinson.

The answer is (D).



Review Time

PROBABILITY

The SAT usually has one or two questions that test probability, which is a measure of the likelihood that an event will occur.

The probability P of an event = $\frac{\text{number of ways event the can occur}}{\text{number of possible events}}$.

Finding the probability of an event is a 2-step process:

- * First, count the total number of **possible** events.
- * Second, count how many ways the **desired** event can happen.

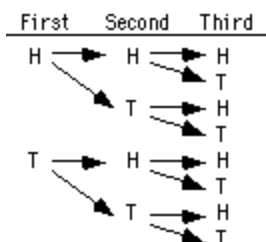
For example, find the probability of tossing a penny three times and getting "heads" exactly one time.

Think about this problem before you read the solution...

Step 1: Count the total number of possible events.

If you toss a penny three times, each time you might get a head or a tail.

There are 8 possible outcomes as shown in the table below:



Step 2: Count the number of ways the desired event can occur.

There are 3 ways to get "heads" exactly one time: HTT, THT, or TTH.

Therefore the probability $P = 3/8$

Studyguide for the SAT Skill Review: Probability

Note the following:

A probability cannot be greater than 1.

From the above probability tree, answer the following.

1. What is the probability of tossing exactly 3 heads?
2. What is the probability of tossing exactly 2 heads?
3. What is the probability of tossing exactly 1 head?
4. What is the probability of tossing exactly 0 heads?

The answer to 1 is $\frac{1}{8}$.

The answer to 2 is $\frac{3}{8}$.

The answer to 3 is $\frac{3}{8}$.

The answer to 4 is $\frac{1}{8}$.

The sum of the probabilities is 1.

There is another way to find the number of possible outcomes besides using a probability tree. Multiply the number of possible outcomes for each toss. The number of possible outcomes for each toss is 2. The number of tosses is 3. $2 \times 2 \times 2$ equals the total number of outcomes.

Use this to find the total number of outcomes for the following.

Example:

If Sally has 2 coats and 3 scarves, how many different outfits can she create?

Solution:

The number of possible outfits is the same as the number of outcomes. Sally can create 2×3 or 6 different outfits.

Studyguide for the SAT Skill Review: Probability

The probability tree would look like the following:

The two coats are C_1 and C_2 and the three scarves are S_1 , S_2 , and S_3 .

Coat	Scarf	Combination
C_1	S_1	$C_1 S_1$
	S_2	$C_1 S_2$
	S_3	$C_1 S_3$
C_2	S_1	$C_2 S_1$
	S_2	$C_2 S_2$
	S_3	$C_2 S_3$

The easier way to find the number of outcomes
is to multiply the number of coats by the number of scarves.



Question 1

A card is drawn from a standard 52 card deck.

What is the probability that this card is either a 5 or a 9?

- (A) $\frac{1}{52}$
- (B) $\frac{1}{26}$
- (C) $\frac{1}{13}$
- (D) $\frac{2}{13}$
- (E) $\frac{5}{9}$

Question 2

A drawer contains 12 red socks, 14 green socks, and 8 black socks.

A sock is taken out at random. What is the probability that the sock is NOT red?

- (A) $\frac{1}{12}$
- (B) $\frac{6}{17}$
- (C) $\frac{6}{11}$
- (D) $\frac{11}{17}$
- (E) $\frac{11}{6}$

Question 3

An average of 6% of widgets manufactured by ACME Widgets are defective. One lot had 42 defective widgets in it. What is the best estimate of the lot size?

- (A) 25.2
- (B) 70
- (C) 420
- (D) 700
- (E) 17,000

Question 4

Four coins are flipped. What is the probability that there will be exactly two heads?

- (A) $\frac{1}{16}$
- (B) $\frac{1}{8}$
- (C) $\frac{1}{4}$
- (D) $\frac{3}{8}$
- (E) $\frac{1}{2}$

Question 5

Four coins are flipped. What is the probability that there will be at least two heads?

- (A) $\frac{1}{8}$
- (B) $\frac{1}{4}$
- (C) $\frac{1}{2}$
- (D) $\frac{9}{16}$
- (E) $\frac{11}{16}$

Question 6

Each student at East High School is assigned a three-digit student ID.
How many unique three-digit numbers can be created?

- (A) 30
- (B) 100
- (C) 300
- (D) 1,000
- (E) 3,000

Question 7

How many five-digit zip codes can be created if 0 cannot be the first digit of the zip code?

- (A) 900
- (B) 10,000
- (C) 90,000
- (D) 100,000
- (E) 9,000,000

Question 8

For a certain bag of marbles, the probability of drawing a red marble is $\frac{3}{8}$. What is the probability of not drawing a red marble?

- (A) $\frac{1}{8}$
- (B) $\frac{2}{8}$
- (C) $\frac{3}{8}$
- (D) $\frac{1}{2}$
- (E) $\frac{5}{8}$

Question 9

Fifteen students are to be divided into five groups of three students each by drawing numbers from a bag. The bag contains fifteen slips of paper. There are three slips containing each number 1 - 5. If Marjorie is the first student to pick and picks a 2, what is the probability that the person behind her will also pick a 2?

- (A) $\frac{1}{15}$
- (B) $\frac{1}{7}$
- (C) $\frac{2}{15}$
- (D) $\frac{3}{14}$
- (E) $\frac{1}{2}$

Question 10

If the probability of drawing a blue marble out of a bag of 30 marbles is $\frac{2}{5}$, how many blue marbles are in the bag?

(A) 2

(B) 5

(C) 12

(D) 15

(E) 20



Hint for Question 1:

How many 5's are there in a deck?

Hint for Question 2:

What is the total number of socks?

Hint for Question 3:

The number 42 is 6% of what?

Hint for Question 4:

Write all of the possible combinations of outcomes. Use the figure with three coin tosses in the Skill Review. Just add another column to the right to create the possible outcomes when you toss four coins. You should get the following 16 outcomes.

HHHH	HHHT	HHTH	HHTT
HTHH	HTHT	HTTH	HTTT
THHH	THHT	THTH	THTT
TTHH	TTHT	TTTH	TTTT

How many ways are there to get **exactly** two heads?

Hint for Question 5:

Use the list of possible outcomes for tossing four coins from the hint in question 4 to determine how many outcomes have two heads or more. (Having at least two heads means you have two or more heads.)

Hint for Question 6:

This is a combination problem. Find the product of the possible number of digits in each position of the number.

Hint for Question 7:

This is a combination problem. Find the product of the possible number of digits in each position of the number. It is important that 0 cannot be the first digit.

Studyguide for the SAT Skill Quiz A: Probability

Hint for Question 8:

This question can be answered by subtracting.

Hint for Question 9:

Remember that after Marjorie picks a number, there are only 14 numbers left.

Hint for Question 10:

Set up the probability equation.
You are looking for the number of desirable events.



Question 1.

The answer is (D).

Explanation for Question 1:

There are four 5's and four 9's in a deck of cards.

The probability is $\frac{8}{52}$ or $\frac{2}{13}$.
The answer is (D).

Question 2.

The answer is (D).

Explanation for Question 2:

There are 34 socks total.

Of the 34 socks 22 are not red.

The probability is $\frac{22}{34} = \frac{11}{17}$.

The answer is (D).

Question 3.

The answer is (D).

Explanation for Question 3:

$\frac{42}{\text{of}} = \frac{6\%}{100}$

$\text{of} = 4200 \div 6 = 700$

700 is a good estimate of the lot size.

The answer is (D).

Question 4.

The answer is (D).

Explanation for Question 4:

There are 6 ways to get exactly two heads with four coins.

HHTT

HTHT

HTTH

THHT

THTH

TTHH

and 16 possible combinations total. $P = \frac{6}{16} = \frac{3}{8}$

The answer is (D).

Question 5.

The answer is (E).

Explanation for Question 5:

There are six ways to get two heads (see the previous problem).

There are four ways to get three heads.

HHHT

HTHH

HHTH

THHH

and one way to get 4 heads HHHH.

There are 11 ways to get two or more heads and 16 total.

$$P = \frac{11}{16}$$

The answer is (E).

Studyguide for the SAT Skill Quiz A: Probability

Question 6.

The answer is (D).

Explanation for Question 6:

The number of three-digit numbers that can be created is the product of the possible digits in each position of the number.

$$\begin{array}{ccc} \overline{\quad} & \overline{\quad} & \overline{\quad} \\ 0-9 & 0-9 & 0-9 \end{array}$$

There are 10 possible digits that can go in each position of the number.

$$\text{Combination} = 10 \times 10 \times 10 = 1000$$

The answer is (D).

Question 7.

The answer is (C).

Explanation for Question 7:

The number of five-digit numbers that can be created is the product of possible digits in each position of the number.

$$\begin{array}{ccccc} \overline{\quad} & \overline{\quad} & \overline{\quad} & \overline{\quad} & \overline{\quad} \\ 1-9 & 0-9 & 0-9 & 0-9 & 0-9 \end{array}$$

There are 10 possible digits that can go in the last four positions of the number. There are only 9 digits that can go in the first position of the number.

$$\text{Combination} = 9 \times 10 \times 10 \times 10 \times 10 = 90,000$$

The answer is (C).

Question 8.

The answer is (E).

Explanation for Question 8:

The probability that the marble will not be red is

$$1 - \frac{3}{8} = \frac{5}{8}$$

The answer is (E).

Studyguide for the SAT Skill Quiz A: Probability

Question 9.

The answer is (B).

Explanation for Question 9:

There are 14 possible events now that one slip of paper has been chosen.

There are two desirable events now that one 2 has been chosen.

$$\text{Probability} = \frac{2}{14} = \frac{1}{7}$$

The answer is (B).

Question 10.

The answer is (C).

Explanation for Question 10:

There are 30 possible events. Let x = the number of desired events.

$$\text{Probability} = \frac{x}{30} = \frac{2}{5}$$

Cross-multiply:

$$5x = 60$$

Divide by 5:

$$x = 12$$

The answer is (C).

The other option is to try each answer choice until

you get a probability of $\frac{2}{5}$.

Start in the middle with answer (C). If your answer is too small, try answer (D).

If your answer is too large, try answer (B).

$$\text{Answer (C)} \frac{12}{30} = \frac{2}{5}$$



Question 1

A bag contains five red marbles and three green marbles.
What is the probability of drawing a green marble from the bag?

- (A) $1/5$
- (B) $3/8$
- (C) $5/8$
- (D) $3/5$
- (E) 1

Question 2

From a standard 52 card deck, what is the probability of drawing a six?

- (A) $1/52$
- (B) $1/26$
- (C) $3/26$
- (D) $1/13$
- (E) $6/13$

Question 3

A bag has 30 red jelly beans, 40 green jelly beans, and 20 yellow jelly beans.
What is the probability of drawing a red jelly bean?

- (A) $\frac{2}{9}$
- (B) $\frac{1}{3}$
- (C) $\frac{4}{9}$
- (D) $\frac{2}{3}$
- (E) $\frac{3}{4}$

Question 4

Amanda has four blouses and three skirts.
How many different blouse-and-skirt combinations can Amanda create?

- (A) 7
- (B) 12
- (C) 16
- (D) 64
- (E) 81

Question 5

From a standard 52 card deck,
what is the probability of drawing a red jack?

- (A) $1/52$
- (B) $1/26$
- (C) $3/26$
- (D) $1/13$
- (E) $6/13$

Question 6

A fair coin* is tossed three times.

What is the probability that all three times the outcome is a head?

*A fair coin is a coin where the
probability of tossing
a head and a tail is the same.

- (A) $1/8$
- (B) $1/6$
- (C) $3/8$
- (D) $1/4$
- (E) 1

Question 7

A fair coin is tossed three times.

What is the probability that at least two heads were tossed?

- (A) $1/8$
- (B) $1/4$
- (C) $3/8$
- (D) $1/2$
- (E) 1

Question 8

Rasheed has 3 pairs of brown socks, 2 pairs of blue socks and 5 pairs of black socks in his dresser drawer.

If he reaches in his dresser drawer without looking, what is the probability that he will pick a blue pair of socks?

- (A) $1/5$
- (B) $3/10$
- (C) $2/5$
- (D) $1/2$
- (E) $4/5$

Question 9

Tony has 2 shirts, 3 pairs of pants, and 4 sports coats.
How many different outfits can Tony create?

- (A) 2
- (B) 6
- (C) 9
- (D) 12
- (E) 24

Question 10

Tanisha figures she can create 72 different outfits if
she mixes her skirts, blouses, and pairs of shoes.
Tanisha has four skirts and three blouses.
She has how many pairs of shoes?

- (A) 6
- (B) 7
- (C) 12
- (D) 18
- (E) 2

Studyguide for the SAT Skill Quiz B: Probability



Hint for Question 1:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

Find the total number of marbles for the denominator.

The numerator is the number of green marbles.

Hint for Question 2:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

The denominator is the number of cards.

The numerator is the number of 6's in a deck of cards.

Hint for Question 3:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

Find the total number of jelly beans in the bag.

How many of the jelly beans are red?

Use these numbers to get the probability
of drawing a red jelly bean.

Hint for Question 4:

If you are having trouble finding the solution
reread the skill review.

The number of different combinations
is the same as finding the number of outcomes.

Hint for Question 5:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

If you don't play cards much, there are two red jacks.

Studyguide for the SAT Skill Quiz B: Probability

Hint for Question 6:

You may have to make a probability tree.
You should practice making one on your own
but you can use the tree in the skill review.

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

Hint for Question 7:

You may have to make a probability tree.
You should practice making one on your own
but you can use the tree in the lesson.

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

How many outcomes have at least two heads?

Hint for Question 8:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

Find the total number of pairs of socks in the drawer.
How many of the pairs are blue?

Hint for Question 9:

The number of outfits that Tony can create is
the product of his number of shirts, pants,
and sports coats.

Hint for Question 10:

To find the number of different outfits Tanisha can
create, you multiply the number of skirts,
blouses, and pairs of shoes.
Write an equation to represent this and
let her number of shoes be a variable.

Studyguide for the SAT Skill Quiz B: Probability



Question 1.

The answer is (B).

Explanation for Question 1:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

The total number of marbles is $5 + 3 = 8$.

The number of green marbles is 3.

The probability of drawing a green marble is $3/8$.

The answer is (B).

Question 2.

The answer is (D).

Explanation for Question 2:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

There are four 6's in a deck of cards.

The total number of cards in the deck is 52.

The probability of drawing a six is $4/52 = 1/13$.

The answer is (D).

Question 3.

The answer is (B).

Explanation for Question 3:

The total number of jelly beans in the bag is $30 + 40 + 20 = 90$.

The number of jelly beans that are red = 30

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

The probability of drawing a red jelly bean is $30/90 = 1/3$.

The answer is (B).

Studyguide for the SAT Skill Quiz B: Probability

Question 4.

The answer is (B).

Explanation for Question 4:

The number of combinations is the product of the number of blouses and skirts.

The product is $4 \times 3 = 12$.

The number of combinations is 12.

The answer is (B).

Question 5.

The answer is (B).

Explanation for Question 5:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

The number of red jacks is 2.

The total number of cards in a deck is 52.

The probability of drawing a red jack is $2/52 = 1/26$.

The answer is (B).

Question 6.

The answer is (A).

Explanation for Question 6:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

First	Second	Third	Outcome
Toss	Toss	Toss	
H	H	H	HHH
		T	HHT
	T	H	HTH
		T	HTT
T	H	H	THH
		T	THT
	T	H	TTH
		T	TTT

The number of ways to get three heads is 1.

The total number of outcomes is 8.

The probability of getting three heads is $1/8$.

The answer is (A).

Studyguide for the SAT Skill Quiz B: Probability

Question 7.

The answer is (D).

Explanation for Question 7:

The probability P of an event = $\frac{\text{number of ways the event can occur}}{\text{number of possible events}}$

First	Second	Third	Outcome
Toss	Toss	Toss	
H	H	H	HHH
		T	HHT
	T	H	HTH
		T	HTT
T	H	H	THH
		T	THT
	T	H	TTH
		T	TTT

There are four times you have at least two heads.

They are HHH, HHT, HTH and THH.

The total number of outcomes is eight.

The probability of tossing at least two heads is $4/8 = 1/2$.

The answer is (D).

Question 8.

The answer is (A).

Explanation for Question 8:

The total number of pairs of socks in the drawer is $3 + 2 + 5 = 10$.

The number of pairs that are blue is 2.

The probability of drawing a blue pair from the drawer is $2/10 = 1/5$.

The answer is (A).

Question 9.

The answer is (E).

Explanation for Question 9:

The number of outfits that Tony can create is

the product of his number of shirts, pants, and sports coats.

The number of outfits equals $2 \times 3 \times 4 = 24$.

The answer is (E).

Question 10.

The answer is (A).

Explanation for Question 10:

If x represents the number of pairs of shoes,
then the total number of outfits Tanisha can create is $4(3)x = 12x$.

The number given for her total number of outfits is 72.

The equation that represents this information is the following:

$$12x = 72$$

$x = 6$. The number of pairs of shoes is 6.

The answer is (A).



Question 1

What is the probability of rolling an even number with a single six-sided die?

- (A) $\frac{1}{6}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{3}$
- (E) $\frac{1}{2}$

Question 2

If you roll a pair of fair six-sided dice,
what is the probability that the sum will be 5?

- (A) $\frac{5}{12}$
- (B) $\frac{5}{36}$
- (C) $\frac{1}{9}$
- (D) $\frac{1}{18}$
- (E) $\frac{1}{36}$

Question 3

If you are rolling a pair of fair six-sided dice,

which of the following sums has a probability of $\frac{1}{18}$ of being rolled?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Question 4

A card is drawn from a regular 52-card deck.

What is the probability that the card will be a red face card?

(A) $\frac{3}{13}$

(B) $\frac{3}{26}$

(C) $\frac{1}{13}$

(D) $\frac{1}{26}$

(E) $\frac{1}{52}$

Question 5

Billy draws a card from a regular 52-card deck.
Without replacing the first card, he draws a second card.
If the first card that he drew was a king of clubs,
what is the probability that the second card drawn is a face card?

- (A) $\frac{1}{17}$
- (B) $\frac{4}{17}$
- (C) $\frac{3}{13}$
- (D) $\frac{11}{51}$
- (E) $\frac{11}{52}$

Question 6

A bag contains 26 blocks that each have a letter of the alphabet written on it. If Anthony draws one letter, what is the probability that the letter will be a letter in the word Anthony?

- (A) $\frac{1}{26}$
- (B) $\frac{1}{13}$
- (C) $\frac{3}{13}$
- (D) $\frac{7}{26}$
- (E) $\frac{1}{7}$

Question 7

The Roberts family entered a raffle. If each of the 5 members of the family entered one time and there were 250 total entries, what is the probability that a member of the Roberts family wins the raffle?

- (A) $\frac{1}{250}$
- (B) $\frac{1}{125}$
- (C) $\frac{1}{50}$
- (D) $\frac{1}{25}$
- (E) $\frac{1}{5}$

Question 8

A bag of marbles contains five red marbles, 12 blue marbles, and eight green marbles. If one marble is drawn from the bag, what is the probability that the marble will not be green?

- (A) $\frac{17}{25}$
- (B) $\frac{8}{25}$
- (C) $\frac{1}{8}$
- (D) $\frac{1}{17}$
- (E) $\frac{1}{25}$

Question 9

A bag contains marbles numbered 1 through 100.

If one marble is drawn at random, what is the probability that the number drawn will be divisible by 5?

- (A) $\frac{1}{100}$
- (B) $\frac{1}{80}$
- (C) $\frac{1}{20}$
- (D) $\frac{1}{10}$
- (E) $\frac{1}{5}$

Question 10

A bag contains marbles numbered 1 through 100. Zoe draws a marble. Then, without replacing the marble, she draws a second marble. If the first marble that she drew was numbered 5, what is the probability that the second marble drawn is a number from 1 to 10?

- (A) $\frac{1}{8}$
- (B) $\frac{1}{9}$
- (C) $\frac{1}{10}$
- (D) $\frac{1}{11}$
- (E) $\frac{1}{12}$

Studyguide for the SAT Skill Quiz C: Probability



Hint for Question 1:

Remember that the probability is the number of ways the desired event can occur divided by the number of possible events.

Hint for Question 2:

When two fair dice are rolled there are 36 possible outcomes.

Hint for Question 3:

When two fair dice are rolled there are 36 possible outcomes. The larger the sum, the more ways that it could be rolled.

Hint for Question 4:

Remember that the probability is the number of ways the desired event can occur divided by the number of possible events.
How many red face cards are there?

Hint for Question 5:

Remember that because one card has been drawn, there are only 51 cards left.

Hint for Question 6:

Be careful. While the word Anthony contains seven letters, how many different letters are there in the word?

Hint for Question 7:

Remember that the probability is the number of ways the desired event can occur divided by the number of possible events.

Hint for Question 8:

Read the problem carefully. How many total marbles are there? How many are not green?

Hint for Question 9:

How many numbers from 1 to 100 are divisible by 5?

Hint for Question 10:

Remember that after one marble is drawn,
there are only 99 possible events.



Question 1.

The answer is (E).

Explanation for Question 1:

There are six possible numbers that could be rolled:

1, 2, 3, 4, 5, 6

There are three even numbers that could be rolled:

2, 4, 6

$$\text{probability} = \frac{3}{6} = \frac{1}{2}$$

The answer is (E).

Question 2.

The answer is (C).

Explanation for Question 2:

Probability is $\frac{\text{the number of desired events}}{\text{the number of possible events}}$

There are 36 possible outcomes (6×6).

There are four ways that the sum could be 5:

1 and 4; 4 and 1; 2 and 3; 3 and 2

$$\text{Probability} = \frac{4}{36} = \frac{1}{9}$$

The answer is (C).

Studyguide for the SAT Skill Quiz C: Probability

Question 3.

The answer is (B).

Explanation for Question 3:

Probability is $\frac{\text{the number of desired events}}{\text{the number of possible events}}$

There are 36 possible outcomes (6×6).

Examine each answer choice. Because the probability is small, start with the smallest number and work down the list.

Answer (A) There is only one way to roll a 2: 1 and 1

$$\text{Probability} = \frac{1}{36}$$

Answer (B) There are two ways to roll a 3: 1 and 2; 2 and 1

$$\text{Probability} = \frac{2}{36} = \frac{1}{18}$$

The answer is (B).

Question 4.

The answer is (B).

Explanation for Question 4:

There are 52 possible events when a card is drawn.

There are 12 face cards; half of those are red.

There are 6 desired events.

$$\text{Probability} = \frac{6}{52} = \frac{3}{26}$$

The answer is (B).

Question 5.

The answer is (D).

Explanation for Question 5:

There are 51 possible events for the second card drawn.

Because one face card has already been drawn, there are 11 desirable events.

$$\text{Probability} = \frac{11}{51}$$

The answer is (D).

Question 6.

The answer is (C).

Explanation for Question 6:

There are 26 possible events when one letter is drawn.

There are 6 desirable events. (Note that there are 6 different letters in the word Anthony. The n appears twice.)

$$\text{Probability} = \frac{6}{26} = \frac{3}{13}$$

The answer is (C).

Question 7.

The answer is (C).

Explanation for Question 7:

There are 250 possible events and 5 desirable events.

$$\text{Probability} = \frac{5}{250} = \frac{1}{50}$$

The answer is (C).

Question 8.

The answer is (A).

Explanation for Question 8:

There are 25 possible events.

(That is the total number of marbles.)

There are 17 desirable events.

(That is the number of marbles that are not green.)

$$\text{Probability} = \frac{17}{25}$$

The answer is (A).

Question 9.

The answer is (E).

Explanation for Question 9:

There are 100 possible events.

There are 20 desirable events because there are 20 numbers from 1 to 100 that are divisible by 5.

5, 10, 15, 20, ..., 95, 100

$$\text{probability} = \frac{20}{100} = \frac{1}{5}$$

The answer is (E).

Question 10.

The answer is (D).

Explanation for Question 10:

There are 99 possible events after one marble is drawn.

Because the first marble was a 5, there are only 9 desired events.

(There are only 9 numbers between 1 and 10 remaining in the bag.)

$$\text{Probability} = \frac{9}{99} = \frac{1}{11}$$

The answer is (D).

Math Lesson #2

Number Theory

- *Signed Numbers*
- *Even and Odd Numbers*
- *Prime Numbers*

Triumph College Admissions



Skill Lesson

Quiz Time!

A B C ☒ E



Hints!

A B C ☒ E



Answers!

A B C ☒ E



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SIGNED NUMBERS/ABSOLUTE VALUE

Signed Numbers

You can avoid missing questions involving signed numbers by remembering a few simple rules.

Multiplication

positive \times positive	Multiply and the answer is positive.
negative \times negative	Multiply and the answer is positive.
negative \times positive	Multiply and the answer is negative.

Addition

positive + positive	Add and the answer is positive.
negative + negative	Add and the answer is negative.
negative + positive	Take the absolute value of each, subtract the smaller from the larger, and keep the sign of whichever absolute value was larger.

Subtraction

Definition of Subtraction: $a - b = a + (-b)$; a minus b equals a plus the *opposite* of b.

Other Rules

$x > 0$ means that x is a positive number

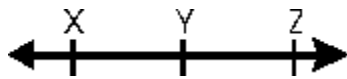
$x < 0$ means that x is a negative number

$$-(a - b) = -a + b = b - a$$

$$(-x)^2 = x^2$$

$$\text{if } x \neq 0, x^2 > 0$$

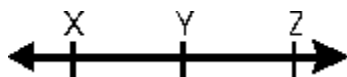
Studyguide for the SAT Skill Review: Signed Numbers/Absolute Value



If, on a number line, a number is to the left of another, it is the smaller number.

That means on the number line above $X < Y$ and $Y < Z$.

Use the above number line to determine if each of the following is positive or negative. One technique you can use to help is to make up specific numbers that fit the problem. For example let $X = -7$, $Y = -2$, and $Z = 3$. Make sure you choose some negative numbers when substituting.

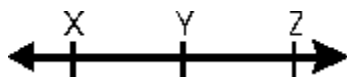


Example 1:

$$Y - X$$

Solution:

Positive, y is greater than x. $-2 - (-7) = -2 + 7 = 5$



Example 2:

$$Z - Y$$

Solution:

Positive, z is greater than y. $3 - (-2) = 3 + 2 = 5$

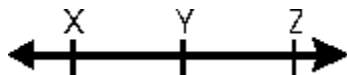


Example 3:

$$X - Y$$

Solution:

Negative, x is less than y. $-7 - (-2) = -7 + 2 = -5$

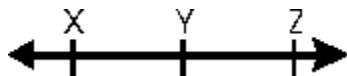


Example 4:

$Z - X$

Solution:

Positive, z is greater than x. $3 - (-7) = 3 + 7 = 10$

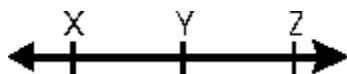


Example 5:

$X - Z$

Solution:

Negative, x is less than z. $-7 - 3 = -10$



Example 6:

Use the same number line. Which of the following represents the largest number?

- (A) $Y - X$
- (B) $Z - X$
- (C) $X - Z$
- (D) $Z - Y$
- (E) $X - Y$

Solution:

We could use the same numbers we used for the previous examples to arrive at the conclusion that B is the largest number.

Example 7:

If $x = 2$ and $y = 3$, then $-(3x - 4y) = ?$

Solution:

$$-(3 \times 2 - 4 \times 3) = -(6 - 12) = -(-6) = 6$$

Absolute Value**Definition of Absolute Value**

The definition has two parts.

The first part is for non-negative numbers. (Positive numbers and zero or $x \geq 0$)

The second part is for negative numbers. ($x < 0$)

Part 1

If $x \geq 0$, then $|x| = x$

Part 2

If $x < 0$, then $|x| = -x$

It is best to read $-x$ here as "the opposite of x " and not as negative x .

From the definition you can see that the absolute value of a number is always positive or 0.

Example 1:

Simplify $|5|$.

Solution:

Which part of the definition would you use for this example?

Because 5 is greater than 0 you use part 1. So $|5| = 5$.

Example 2:

Simplify $|-3|$.

Solution:

Which part of the definition would you use for this example?

Because -3 is less than 0, you use part 2. So $|-3| = -(-3) = 3$.

In this part x equals a negative 3. The absolute value of a negative 3 is the opposite of a negative 3. The opposite of a negative 3 is a positive 3.

Example 3:

Simplify $3|-4| - |5|$.

Solution:

Simplify the absolute values first and then use order of operations.

Step 1. $3(4) - (5)$

Step 2. $12 - 5 = 7$

Absolute value also can be used to indicate distance.

You do not assign a negative sign to distance.

On the number line, the distance between a and b is $|a - b|$ or $|b - a|$.

Because you do not know which value (a or b) is greater, the use of the absolute value ensures the distance will be positive.

Example 4:

What is the distance between -3 and -10?

Solution:

The distance is $|(-3) - (-10)| = |-3 + 10| = |7| = 7$ or

$$|(-10) - (-3)| = |-10 + 3| = |-7| = 7.$$

Example 5:

What points on the number line are 5 units away from 3?

Solution:

There are two answers to this question.

Let x represent the point that is 5 units away from 3.

$$|x - 3| = 5$$

You can get two different equations to solve. The distance is always positive but the number inside the absolute value could be either positive or negative.

If the number inside the absolute value is positive then

$$|x - 3| = x - 3 = 5$$

Solving for x , you get $x = 8$.

If the number inside the absolute value is negative then

$$|x - 3| = -(x - 3) = -x + 3 = 5$$

Solving for x , you get $-x = 2$ and then $x = -2$.

There are two solutions to the problem: 8 and -2.

The check would be $|8 - 3| = 5$ and $|-2 - 3| = |-5| = 5$.

Absolute Value Properties**Example**

Property 1. $|ab| = |a| \times |b|$

$$|-2(3)| = |-2| \times |3| = 2 \times 3 = 6$$

Property 2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$$\left|\frac{-10}{5}\right| = \frac{|-10|}{|5|} = \frac{10}{5} = 2$$



Question 1

If $x^2 - 3x + 4 > x^2 + 3x + 4$, which of the following describes x ?

- (A) $x < 0$
- (B) $x = 0$
- (C) $x > 0$
- (D) $x = 2$
- (E) $x > 2$

Question 2

$|4 - 6| \times |6 - 4| =$

- (A) -4
- (B) 0
- (C) 4
- (D) 20
- (E) 100

Question 3

$6 - (-10) =$

- (A) 4
- (B) 16
- (C) -4
- (D) -16
- (E) 10

Question 4

$$8 + (-3) =$$

- (A) 11
- (B) -5
- (C) -11
- (D) 5
- (E) 2

Question 5

$$-12 - (-4) =$$

- (A) 16
- (B) 8
- (C) -8
- (D) -16
- (E) 3

Question 6

$$(-5) \times (-10) =$$

- (A) 2
- (B) -50
- (C) 50
- (D) -2
- (E) 15

Question 7

$$483 \div (-21) =$$

- (A) 27
- (B) 23
- (C) -23
- (D) -27
- (E) 13

Question 8

$$\frac{-10 \times 31}{5} =$$

- (A) -60
- (B) 62
- (C) -62
- (D) 60
- (E) 52

Question 9

$$\frac{-16 - 10}{13} =$$

- (A) 6
- (B) -26
- (C) 2
- (D) 26
- (E) -2

Question 10

$$-7[4 + (-9)] =$$

- (A) -91
- (B) 35
- (C) 91
- (D) -35
- (E) 17

**Hint for Question 1:**

Eliminate similar quantities on both sides.

x^2 and 4 should be crossed out from both sides leaving $-3x > 3x$.

Add $3x$ to both sides.

Hint for Question 2:

Remember the absolute value is always positive or zero.

Hint for Question 3:

$$6 - (-10) = 6 + (+10)$$

Hint for Question 4:

When adding two numbers of different signs, subtract and use the sign of the larger number.

Hint for Question 5:

$$-12 - (-4) = -12 + (+4)$$

Hint for Question 6:

The product of two like-signed numbers is positive.

Hint for Question 7:

$$483 = (-21) \times (?)$$

The unknown number must be negative to obtain a positive product. This eliminates choices (A), (B), and (E).

Hint for Question 8:

Multiplication and division of two numbers with different signs will give a negative number.

$$5 \times (?) = -310$$

Hint for Question 9:

When adding numbers of the same sign, add and keep the sign.

$$-16 - 10 = -16 + -10$$

$\frac{-26}{13}$ when simplified.

The answer will be negative.

Hint for Question 10:

When adding numbers of different signs, subtract and use the sign of the number with largest absolute value.



(A) (B) (C) (D) (E)

Question 1.

The answer is (A).

Explanation for Question 1:

$$\begin{array}{rclcl} x^2 - 3x + 4 & > & x^2 + 3x + 4 \\ -3x & > & 3x \\ 0 & > & 6x \\ 0 & > & x \\ x & < & 0 \end{array}$$

The answer is (A).

Question 2.

The answer is (C).

Explanation for Question 2:

$$|4 - 6| = |-2| = 2$$

$$|6 - 4| = |2| = 2$$

$$2 \times 2 = 4$$

The answer is (C).

Question 3.

The answer is (B).

Explanation for Question 3:

$$6 - (-10) = 6 + 10 = 16$$

The answer is (B).

Question 4.

The answer is (D).

Explanation for Question 4:

$$8 + (-3) = 5$$

(When adding two numbers with different signs: subtract and use the sign of the larger number.)

The answer is (D).

Question 5.

The answer is (C).

Explanation for Question 5:

$$-12 - (-4) = -12 + 4 = -8$$

The answer is (C).

Question 6.

The answer is (C).

Explanation for Question 6:

$$(-5) \times (-10) = 50$$

The product of two like-signed numbers is positive.

The answer is (C).

Question 7.

The answer is (C).

Explanation for Question 7:

$(-21) \times (?)$ will result in an end digit of 3.

The answer is (C).

$$483 \div (-21) = -23$$

(Division of two numbers with different signs will give a negative number.)

Question 8.

The answer is (C).

Explanation for Question 8:

$$\frac{-10 \times 31}{5} = \frac{-310}{5} = -62$$

The answer is (C).

Question 9.

The answer is (E).

Explanation for Question 9:

$$\frac{-16 - 10}{13} = \frac{-16 + -10}{13} = \frac{-26}{13} = -2$$

The answer is (E).

Question 10.

The answer is (B).

Explanation for Question 10:

$$-7[4 + (-9)] = -7[-5] = 35$$

The answer is (B).



Question 1

Simplify $|-3| + 2 - |-7|$.

- (A) -8
- (B) -6
- (C) -2
- (D) 6
- (E) 12

Question 2

Simplify $2|-5|$.

- (A) -10
- (B) -7
- (C) -3
- (D) 7
- (E) 10

Question 3

Simplify $|-8| - (-8)$.

- (A) -64
- (B) -16
- (C) 0
- (D) 16
- (E) 64

Question 4

Simplify $-|-6| + (-6)$.

- (A) -36
- (B) -12
- (C) 0
- (D) 12
- (E) 36

Question 5

On a number line, which of the following numbers is 4 units from 2?

- (A) -3
- (B) -2
- (C) 1
- (D) 2
- (E) 3

Question 6

Simplify $|-3| + |6|$.

- (A) -18
- (B) -9
- (C) -3
- (D) 3
- (E) 9

Question 7

Simplify $-2|-4| + (-3)$.

- (A) 11
- (B) 5
- (C) -5
- (D) -9
- (E) -11

Question 8

Simplify $\frac{-10}{|2|}$.

- (A) -12
- (B) -8
- (C) -5
- (D) 5
- (E) 8

Question 9

Simplify $\frac{-10}{|-2|}$.

- (A) -12
- (B) -8
- (C) -5
- (D) 5
- (E) 8

Question 10

Simplify $|7| - |-2|$.

- (A) 9
- (B) 5
- (D) -5
- (D) -9
- (E) -14

Studyguide for the SAT Skill Quiz B: Signed Numbers/Absolute Value



Hint for Question 1:

Simplify the absolute values first. Be careful of the negative sign outside the absolute value sign.

Hint for Question 2:

Simplify the absolute value first. Remember an absolute value is always positive.

Hint for Question 3:

Simplify the absolute value first and remember the opposite of a negative is a positive.
 $-(-8) = 8$

Hint for Question 4:

Simplify the absolute value first. Do not be tempted to distribute the first negative sign. Remember how to add two negative numbers.

Hint for Question 5:

On a number line, a number can be 4 units from 2 either to the left or to the right. There are two numbers that are 4 units from 2.

Hint for Question 6:

Simplify the absolute values first.

Hint for Question 7:

Simplify the absolute value first. Remember that a negative times a positive is a negative.

Hint for Question 8:

Because both numerator and denominator are absolute values you have two ways to solve the problem. Either simplify the absolute values first or divide and take the absolute value of the quotient. It is best to remain consistent and because the method used in the previous problems was to simplify the absolute values first, you should continue to use this method.

Hint for Question 9:

Simplify the absolute values first. Continue to remain consistent.

Hint for Question 10:

Simplify the absolute values first.



Question 1.

The answer is (C).

Explanation for Question 1:

Simplify the absolute values first.

$$|-3| + 2 - |-7| =$$

$$3 + 2 - 7 = -2 \text{ The answer is (C).}$$

Question 2.

The answer is (E).

Explanation for Question 2:

$$2|-5| =$$

$$2(5) = 10 \text{ The answer is (E).}$$

Question 3.

The answer is (D).

Explanation for Question 3:

$$|-8| - (-8) =$$

$$8 + 8 = 16 \text{ The answer is (D).}$$

Question 4.

The answer is (B).

Explanation for Question 4:

$$-|-6| + (-6) =$$

$$-6 + (-6) = -12 \text{ The answer is (B).}$$

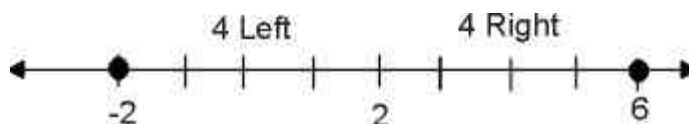
Question 5.

The answer is (B).

Explanation for Question 5:

If the number is 4 units from 2 to the right, then add 4 to 2. $4 + 2 = 6$

If the number is 4 units from 2 to the left, then subtract 4 from 2. $2 - 4 = -2$.



The two solutions are 6 and -2. From the given answer choices -2 is correct. The answer is (B).

Question 6.

The answer is (E).

Explanation for Question 6:

$$|-3| + |6| =$$

$$3 + 6 = 9 \text{ The answer is (E).}$$

Question 7.

The answer is (E).

Explanation for Question 7:

$$-2|-4| + (-3) =$$

$$-2(4) + (-3) =$$

$$-8 + (-3) = -11 \text{ The answer is (E).}$$

Question 8.

The answer is (D).

Explanation for Question 8:

Simplify the absolute values first.

$$\frac{|-10|}{|-2|} =$$

$$\frac{10}{2} = 5 \text{ The answer is (D).}$$

Question 9.

The answer is (C).

Explanation for Question 9:

$$\frac{-|10|}{|2|} =$$

$$\frac{-(10)}{2} = -5 \text{ The answer is (C).}$$

Question 10.

The answer is (B).

Explanation for Question 10:

$$|7| - |-2| =$$

$$7 - 2 = 5 \text{ The answer is (B).}$$



Question 1

If $a > 0$ and $b < 0$, which of the following expressions is always negative?

- (A) $a + b$
- (B) $a - b$
- (C) $a^2 + b^2$
- (D) $3ab$
- (E) $a(b + 2)$

Question 2

If x and y are both negative integers, which of the following will always be a positive integer?

- (A) $x + y$
- (B) $x - y$
- (C) $y - x$
- (D) xy
- (E) $\frac{x}{y}$

Question 3

Which of the following expressions is equivalent to $|r - s|$?

- (A) rs
- (B) $r - s$
- (C) $r + s$
- (D) $|r + s|$
- (E) $|s - r|$

Question 4

What is the value of $3 - 2|-4 + 3^2|$?

- (A) -23
- (B) -7
- (C) -1
- (D) 1
- (E) 5

Question 5

The temperature at 7 a.m. was -23 degrees Celsius.

By noon the temperature had risen 13 degrees Celsius before dropping 17.5 degrees Celsius by 8 p.m.

What was the temperature in degrees Celsius at 8 p.m.?

- (A) -27.5
- (B) -18.5
- (C) -4.5
- (D) 7.5
- (E) 18.5

Question 6

If $a < 0 < b$, then which of the following expressions is equal to $|a - b| + |a + b|$?

- (A) $2a + 2b$
- (B) $2a - 2b$
- (C) $2a$
- (D) $2b$
- (E) 0

Question 7

What is the value of $-3[2 - 5(7 - 9)]$?

- (A) -36
- (B) -12
- (C) 12
- (D) 24
- (E) 36

Question 8

Which of the following expressions could be used to find the distance between two numbers a and b on a number line?

- (A) $a + b$
- (B) $a - b$
- (C) ab
- (D) $|a + b|$
- (E) $|a - b|$

Question 9

What is the value of $|3(2 - 6)^2 - 50|$?

- (A) -102
- (B) -2
- (C) 2
- (D) 94
- (E) 98

Question 10

If $x < 0 < y$, then which of the following is not necessarily negative?

- (A) $x^3 y$
- (B) $2x - y$
- (C) $\frac{3x}{4y}$
- (D) $5(x + y)$
- (E) $xy + x$



Hint for Question 1:

Pick numbers for a and b.
You may need to pick numbers more than once to eliminate all answer choices but one.

Hint for Question 2:

Read the problem carefully.
The answer must always be positive and an integer.
An integer is a positive or negative whole number.
(...-3, -2, -1, 0, 1, 2, 3, ...)

Hint for Question 3:

Pick numbers for r and s.
You may need to pick numbers more than once to eliminate all answer choices but one.

Hint for Question 4:

Remember the order of operations!
Simplify inside of the absolute value first.

Hint for Question 5:

Though this problem looks long,
it is a basic addition problem.

Hint for Question 6:

Choose numbers for a and b. Notice that a must be less than zero and b must be greater than zero.

Hint for Question 7:

Remember the order of operations.
Simplify inside the parentheses first.

Hint for Question 8:

Draw a number line. Choose numbers for a and b.
Remember that distance is always positive.

Hint for Question 9:

Remember the order of operations.
Everything inside the absolute
value should be simplified first.

Hint for Question 10:

Choose numbers for x and y . You are looking for the answer choice
that is not always negative. When you get an answer choice that is
positive or zero, you have the answer.



Question 1.

The answer is (D).

Explanation for Question 1:

Let $a = 1$ and $b = -1$.

(A) $1 + -1 = 0$ (A) cannot be the answer.

(B) $1 - (-1) = 1 + 1 = 2$ (B) cannot be the answer.

(C) $1^2 + (-1)^2 = 1 + 1 = 2$ (C) cannot be the answer.

(D) $3(1)(-1) = -3$ (D) could be the answer.

(E) $1(-1 + 2) = 1(1) = 1$ (E) cannot be the answer.

The answer is (D).

Question 2.

The answer is (D).

Explanation for Question 2:

Let $x = -1$ and $y = -2$.

(A) $-1 + -2 = -3$; (A) cannot be the answer.

(B) $-1 - (-2) = -1 + 2 = 1$; (B) could be the answer.

(C) $-2 - (-1) = -2 + 1 = -1$; (C) cannot be the answer.

(D) $-1(-2) = 2$;

(D) could be the answer.

(E) (E) cannot be the answer. (is not an integer.)

Let $x = -2$ and $y = -1$.

(B) $-2 - (-1) = -2 + 1 = -1$; (B) cannot be the answer.

The answer is (D).

Question 3.

The answer is (E).

Explanation for Question 3:

Let $r = 3$ and $s = 1$.

(A) $3(1) = 3$; (A) cannot be the answer.

(B) $3 - 1 = 2$; (B) could be the answer.

(C) $3 + 1 = 4$; (C) cannot be the answer.

(D) $|3 + 1| = 4$; (D) cannot be the answer.

(E) $|1 - 3| = |-2| = 2$; (E) could be the answer.

Let $r = 1$ and $s = 3$.

(B) $1 - 3 = -2$; B cannot be the answer.

The answer is (E). There is no need to try it.

Question 4.

The answer is (B).

Explanation for Question 4:

$$\begin{aligned} 3 - 2|-4 + 3^2| &= 3 - 2|-4 + 9| \\ &= 3 - 2|5| \\ &= 3 - 2(5) \\ &= 3 - 10 \\ &= -7 \end{aligned}$$

The answer is (B).

Question 5.

The answer is (A).

Explanation for Question 5:

The temperature rose 13 degrees from -23 degrees.

$$-23 + 13 = -10$$

Then, the temperature dropped 17.5 degrees.

$$-10 - 17.5 = -27.5$$

The answer is (A).

Question 6.

The answer is (D).

Explanation for Question 6:

Let $a = -2$ and $b = 1$.

$$|a - b| + |a + b| = |-2 - 1| + |-2 + 1| = |-3| + |-1| = 3 + 1 = 4$$

(A) $2(-2) + 2(1) = -4 + 2 = -2$; (A) cannot be the answer

(B) $2(-2) - 2(1) = -4 - 2 = -6$; (B) cannot be the answer

(C) $2(-2) = -4$; (C) cannot be the answer.

(D) $2(1) = 2$; (D) could be the answer.

(E) 0 is not the answer.

The answer is (D).

Question 7.

The answer is (A).

Explanation for Question 7:

$$\begin{aligned} -3[2 - 5(7 - 9)] &= -3[2 - 5(-2)] \\ &= -3(2 - (-10)) \\ &= -3(2 + 10) \\ &= -3(12) \\ &= -36 \end{aligned}$$

The answer is (A).

Question 8.

The answer is (E).

Explanation for Question 8:

Let $a = 1$ and $b = 5$.

a and b are 4 units apart.

(A) $1 + 5 = 6$; (A) cannot be the answer

(B) $1 - 5 = -4$; (B) cannot be the answer
(Distance is always positive.)

(C) $1(5) = 5$; (C) cannot be the answer

(D) $|1 + 5| = |6| = 6$; (D) cannot be the answer

(E) $|1 - 5| = |-4| = 4$

The answer is (E).

Question 9.

The answer is (C).

Explanation for Question 9:

$$|3(2 - 6)^2 - 50|$$

$$= |3(-4)^2 - 50|$$

$$= |3(16) - 50|$$

$$= |48 - 50|$$

$$= |-2|$$

$$= 2$$

The answer is (C).

Question 10.

The answer is (D).

Explanation for Question 10:

Let $x = -1$ and $y = 5$.

(A) $(-1)3(5) = -5$

(B) $2(-1) - 5 = -2 - 5 = -7$

(C) $\frac{3(-1)}{4(5)} = \frac{-3}{20}$

(D) $5(-1 + 5) = 5(4) = 20$

The answer is (D).

If all of the answer choices had been negative, you would have needed to select different numbers for x and y and then try the answer choices again.



EVENS AND ODDS

Even numbers are the set of numbers that are divisible by 2.

$\{\dots-6, -4, -2, 0, 2, 4, 6, \dots\}$. Remember a number is divisible by 2, if it has no remainder when it is divided by 2.

Note: 0 is an even number.

Odd numbers are the set $\{\dots-5, -3, -1, 1, 3, 5, \dots\}$.

Consecutive **even** numbers are two units apart. For example, if x is an **even** number then the next consecutive **even** number is $x + 2$.

Properties of Odd and Even Numbers

To check a property substitute appropriate numbers.

Addition properties of even and odd numbers

<u>Property</u>	<u>Example</u>
even + even = even	$2 + 8 = 10$
odd + odd = even	$3 + 9 = 12$
odd + even = odd	$3 + 8 = 11$

Multiplication properties of even and odd numbers

<u>Property</u>	<u>Example</u>
even \times even = even	$4 \times 6 = 24$
even \times odd = even	$4 \times 5 = 20$
odd \times odd = odd	$3 \times 9 = 27$

Example 1:

If s is an odd integer, what are the next two consecutive odd integers?

Solution:

$s + 2$ and $s + 4$

Example 2:

If x is an odd integer and y is an even integer, tell whether each expression is odd or even.

(A) x^2

(B) xy

(C) y^2

(D) $x + y$

(E) $2x + y$

Solution:

(A) is odd. (B) is even. (C) is even. (D) is odd. (E) is even.



Question 1

If $a + 3$ is an even integer, then a could be which of the following?

- (A) -4
- (B) -2
- (C) 0
- (D) 1
- (E) 2

Question 2

If r is an odd integer, which of the following represents an odd integer?

- (A) $r - 1$
- (B) $r + 1$
- (C) $r + 2$
- (D) $r + 3$
- (E) $r + 5$

Question 3

What is the sum of the even integers between 5 and 15?

- (A) 50
- (B) 49
- (C) 48
- (D) 40
- (E) 38

Question 4

What is the difference between the sum of the even integers between 5 and 20 and the sum of the odd integers between 5 and 20? Do not include the 5 or the 20 in your sums.

- (A) -5
- (B) -7
- (C) 0
- (D) 5
- (E) 7

Question 5

If the product of two consecutive positive even integers is 224, what is the smaller integer?

- (A) 12
- (B) 14
- (C) 16
- (D) 18
- (E) 20

Question 6

In a list of numbers, every odd term (beginning with the first) is 3, and every even term (beginning with the second) is $-1/3$. What is the product of the first six terms in the list?

- (A) -10
- (B) -6
- (C) -1
- (D) 1
- (E) 6

Question 7

What is the sum of the first 10 positive even integers?

- (A) 45
- (B) 55
- (C) 90
- (D) 110
- (E) 112

Question 8

If x is an odd integer greater than 3, what is the next greater even integer?

- (A) $2x$
- (B) x^2
- (C) $x + 3$
- (D) $x + 2$
- (E) $x + 1$

Question 9

$\frac{(N - 2)(N - 4)}{2}$ is an integer if N is equal to

- (A) 1 only
- (B) 2 only
- (C) 6 only
- (D) any odd integer
- (E) any even integer

Question 10

The integer 75 is to be expressed as the sum of n consecutive positive integers. The value of n could be which of the following?

- I. 2
- II. 3
- III. 5

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) I, II, and III



Hint for Question 1:

Use backsolving.

Hint for Question 2:

Pick an odd integer for r .

Hint for Question 3:

List the even integers between 5 and 15.

Hint for Question 4:

List the even integers between 5 and 20 and list the odd integers between 5 and 20.

Hint for Question 5:

Use backsolving.

Hint for Question 6:

Make up the list of numbers.

Hint for Question 7:

List the first 10 positive even integers.

Hint for Question 8:

Pick an appropriate number for x .

Hint for Question 9:

Use backsolving.

Hint for Question 10:

Do you understand what the question asks you to find? Can you find two consecutive integers whose sum is 75? Can you find three consecutive integers whose sum is 75? Can you find five consecutive integers whose sum is 75?



Question 1.

The answer is (D).

Explanation for Question 1:

Substitute in each answer and see which one makes $a + 3$ even. If $a = 1$, then $a + 3 = 4$. The answer is (D).

Question 2.

The answer is (C).

Explanation for Question 2:

Let $r = 3$. Choice (A) = 2. Choice (B) = 4. Choice (C) = 5. Stop! The answer is (C).

Question 3.

The answer is (A).

Explanation for Question 3:

The even integers between 5 and 15 are 6, 8, 10, 12, and 14. Their sum is 50. The answer is (A).

Question 4.

The answer is (B).

Explanation for Question 4:

You could add up the even, add up the odd, and then subtract. The answer is -7. Or think about the following:

6, 8, 10, 12, 14, 16, 18
7, 9, 11, 13, 15, 17, 19

If you subtract the bottom numbers from the top you get -1, seven times. The answer is (B).

Question 5.

The answer is (B).

Explanation for Question 5:

Start backsolving with (C). If 16 is the smallest, the next consecutive positive even integer is 18. $16 \times 18 = 288$. This number is too large so the answer is either (A) or (B). Try (B) next. $14 \times 16 = 224$. The answer is (B).

Question 6.

The answer is (C).

Explanation for Question 6:

The first six terms are $3, -\frac{1}{3}, 3, -\frac{1}{3}, 3, -\frac{1}{3}$. Their product is -1.

The answer is (C).

Question 7.

The answer is (D).

Explanation for Question 7:

The list is 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. The sum is 110. The answer is (D).
Look for some easy ways to add these numbers.

Question 8.

The answer is (E).

Explanation for Question 8:

Let $x = 5$. Then (A) is 10, (B) is 25, (C) is 8, (D) is 7, and (E) is 6. The answer is (E).
(Note: The problem asks for consecutive integers. The consecutive integer after x is $x + 1$).

Question 9.

The answer is (E).

Explanation for Question 9:

For (A), the expression equals $\frac{3}{2}$.

For (B), the expression equals 0. For (C), the expression equals 4.
For (D), try some other odd integers. The numerator always will be odd, so when divided by 2, you always get a fraction. For (E), try other even integers. The numerator is always even, so when divided by 2, you always get an integer. The answer is (E). (Note: you can generalize this one.)

Question 10.

The answer is (E).

Explanation for Question 10:

For number I, begin by dividing 75 by 2. List the consecutive integers near this number. Can the sum of any two consecutive integers equal 75? 36, 37, 38, 39, Yes. $37 + 38 = 75$. For number II, divide 75 by 3. List the consecutive integers near this number. Can the sum of any three consecutive integers equal 75? 24, 25, 26, 27, 28, Yes. $24 + 25 + 26 = 75$. For number III, divide 75 by 5. List the consecutive integers near this number. Can the sum of any five consecutive integers equal 75? 13, 14, 15, 16, 17, 18, 19, Yes. $13 + 14 + 15 + 16 + 17$. The answer is (E).



Question 1

If a , b , and c are consecutive even integers and $a < b < c$, how much greater is $c - a$ than $b - a$?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

Question 2

If every digit of a whole number is either a 5 or a 9, then the number must be which of the following?

- (A) prime
- (B) odd
- (C) divisible by 5
- (D) divisible by 9
- (E) even

Question 3

Which of the following is equal to the sum of two consecutive odd integers?

- (A) 502
- (B) 503
- (C) 504
- (D) 505
- (E) 506

Question 4

If a is an odd integer and b is an even integer, which of the following must be an odd integer?

- (A) ab
- (B) a/b
- (C) b/a
- (D) $a + b/2$
- (E) $ab + a$

Question 5

The product of two consecutive positive integers is always divisible by which of the following?

- (A) 2
- (B) 3
- (C) 4
- (D) 5
- (E) It cannot be determined

Question 6

If $8/x$ is an odd integer, which of the following could be a value of x ?

- (A) $1/5$
- (B) $3/8$
- (C) $2/5$
- (D) $5/8$
- (E) $8/5$

Question 7

If a is an odd integer, which of the following is an odd integer?

- (A) $2a$
- (B) $3a$
- (C) $4a$
- (D) $a + 1$
- (E) $2a + 4$

Question 8

a , b , and c are consecutive odd integers with $a < b < c$.

How much greater is $b + c$ than $a + b$?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

Question 9

Which of the following sets of numbers has the property that the sum of any two numbers in the set is also a number in the set?

- I. The set of odd numbers.
- II. The set of prime numbers.
- III. The set of even numbers.

- (A) I only
- (B) II only
- (C) III only
- (D) I and III
- (E) I, II, and III

Question 10

$\frac{(N - 2)(N - 4) - 1}{2}$ is an integer, if the value of N is which of the following?

- (A) 1 only
- (B) 2 only
- (C) 3 only
- (D) any odd integer
- (E) any even integer



Hint for Question 1:

Pick appropriate numbers for a, b, and c.

Hint for Question 2:

Write out a list of possible numbers.

Hint for Question 3:

If you add two odds the answer must be what type of a number?

Hint for Question 4:

Pick appropriate numbers for a and b.

Hint for Question 5:

Would the product of any two consecutive positive integers be odd or even?

Hint for Question 6:

Use backsolving.

Hint for Question 7:

Pick an appropriate number for a.

Hint for Question 8:

Pick appropriate numbers for a, b, and c.

Hint for Question 9:

Make a list of each set for I, II, and III.

Hint for Question 10:

Use backsolving for the first three choices. Then pick appropriate numbers.



Question 1.

The answer is (C).

Explanation for Question 1:

Let $a = 2$, $b = 4$, and $c = 6$, then $c - a = 4$ and $b - a = 2$ so $c - a$ is 2 greater than $b - a$. Try other numbers to see if this stays true. The answer is (C).

Question 2.

The answer is (B).

Explanation for Question 2:

Possible numbers are 59, 95, 555, 559, 595, 599, etc. Are all prime? No! Are all odd? Yes! Stop! The answer is (B).

Question 3.

The answer is (C).

Explanation for Question 3:

The sum of the two odd numbers must be even. So choices (A), (C), or (E) could be answers. Try numbers for each answer choice until you get the correct answer. Because all are 502 or larger, begin with half of the number

$\frac{502}{2} = 251$. Your possible choices could then be 251, 253, 255, 257.

Add any two consecutive odd numbers from this list to find your answer. The answer is (C).

Question 4.

The answer is (E).

Explanation for Question 4:

Let $a = 3$ and $b = 6$. Choice (A) is 18, choice (B) is $\frac{1}{2}$, choice (C) is 2,

choice (D) is 6, and choice (E) is 21. The answer is (E). (Note: It is possible to get fractions for (B), (C), and (D); therefore, they can be quickly eliminated.)

Question 5.

The answer is (A).

Explanation for Question 5:

The product of any two consecutive positive integers will always be even. The product will always be divisible by 2. The answer is (A).

Question 6.

The answer is (E).

Explanation for Question 6:

Substitute in each answer choice and simplify.

$$(A) \frac{8}{1/5} = 40 \quad (B) \frac{8}{3/8} = \frac{64}{3} \quad (C) \frac{8}{2/5} = 20 \quad (D) \frac{8}{5/8} = \frac{64}{5} \quad (E) \frac{8}{8/5} = 5$$

The answer is (E).

Question 7.

The answer is (B).

Explanation for Question 7:

Let $a = 5$. Choice (A) is $2(5) = 10$. Choice (B) is $3(5) = 15$. Stop!

The answer is (B).

Question 8.

The answer is (E).

Explanation for Question 8:

Let $a = 3$, $b = 5$, and $c = 7$. $b + c = 12$ and $a + b = 8$. The answer is (E).

Question 9.

The answer is (C).

Explanation for Question 9:

$$I = \{\dots -1, 1, 3, 5, \dots\}$$

$$II = \{2, 3, 5, 7, 11, \dots\}$$

$$III = \{\dots -2, 0, 2, 4, 6\}$$

Add any two numbers in I and you don't get another odd. No.

Add any two numbers in II and you don't always get another prime. No.

Add any two numbers in III and you always get another even number. Yes

The answer is (C).

Question 10.

The answer is (D).

Explanation for Question 10:

For choice A the expression equals 1. For choice (B) the expression equals

$-\frac{1}{2}$. For choice (C) the expression equals -1. Because you have integral

answers for (A) and (C) it leads you to believe that (D) may be the answer. Before making your final selection try a few more odd numbers for N just to be sure.

The answer is (D).



PRIME NUMBERS

Definition of a **Prime Number**: An integer greater than 1 is a **prime number** if it has only two positive factors: 1 and itself.

Example: 11 is prime because its only factors are 1 and 11.
9 is not prime because its factors are 1, 3, and 9.

A List of the First Ten Prime Numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Note: 1 is **NOT** a prime number and the smallest and only even prime number is 2.

Prime factorization is the process of expressing a number as a product of prime numbers only.

Example: The **prime factorization** of 24 is $2 \times 2 \times 2 \times 3$ or $2^3 \times 3$.
The **prime factorization** of 15 is 5×3 .

Example 1:

If $xy = 13$ and x and y are positive integers then $x + y = ?$

- (A) 13
- (B) 14
- (C) 20
- (D) 23
- (E) 26

Solution:

Because 13 is a prime number, the only possible values for x and y are 1 and 13. The answer is (B).

Example 2:

What is the sum of the first five prime numbers?

- (A) 18
- (B) 28
- (C) 30
- (D) 34
- (E) 38

Solution:

The first five prime numbers are 2, 3, 5, 7, and 11, and their sum is 28.

The answer is (B).



Question 1

What is the sum of the first six prime numbers?

- (A) 21
- (B) 29
- (C) 31
- (D) 41
- (E) 42

Question 2

What is the product of the two smallest prime numbers?

- (A) 2
- (B) 6
- (C) 10
- (D) 15
- (E) 35

Question 3

What is the largest prime number less than 52?

- (A) 37
- (B) 45
- (C) 47
- (D) 49
- (E) 51

Question 4

What is the prime factorization of 48?

- (A) $2 \times 2 \times 2 \times 3 \times 3$
- (B) $2 \times 2 \times 2 \times 2 \times 3$
- (C) $3 \times 4 \times 4$
- (D) $2 \times 3 \times 2 \times 4$
- (E) 6×8

Question 5

What is the first prime number after 79?

- (A) 81
- (B) 83
- (C) 85
- (D) 87
- (E) 89

Question 6

Which of the following is the square of a prime number?

- (A) 125
- (B) 64
- (C) 16
- (D) 8
- (E) 4

Question 7

The set Q consists of all numbers that are the sum of two consecutive prime numbers. For example, the number 60 is in Q because $29 + 31 = 60$. What is the smallest prime number in Q ?

- (A) 3
- (B) 5
- (C) 7
- (D) 8
- (E) 11

Question 8

What is the sum of the first ten prime numbers?

- (A) 101
- (B) 107
- (C) 109
- (D) 121
- (E) 129

Question 9

What is the largest prime number less than 50?

- (A) 37
- (B) 43
- (C) 45
- (D) 47
- (E) 49

Question 10

What is the sum of the two smallest prime numbers?

- (A) 3
- (B) 5
- (C) 7
- (D) 8
- (E) 10

Studyguide for the SAT Skill Quiz A: Prime Numbers



Hint for Question 1:

Write the list of the first six prime numbers and add them.

Hint for Question 2:

What are the two smallest prime numbers?

Hint for Question 3:

Which answer choices are prime numbers? Do you remember how to find the different factors? One approach to see if a number is prime is to divide it by consecutive prime numbers up to the integer that is closest to the square root of the number.

Hint for Question 4:

Start with any factorization and keep breaking it down until you only have prime numbers in the factorization.

Hint for Question 5:

Which answer choices are prime numbers?

Hint for Question 6:

Write your list of prime numbers and square them.

Hint for Question 7:

Be sure to read the question carefully. Write the list of numbers in Q. The answer is the smallest **Prime Number** in Q.

Hint for Question 8:

What are the first ten prime numbers?

Hint for Question 9:

Of the choices, which is the largest number whose only factors are 1 and itself?

Hint for Question 10:

What are the two smallest prime numbers?

Studyguide for the SAT Skill Quiz A: Prime Numbers



Question 1.

The answer is (D).

Explanation for Question 1:

The first six prime numbers are 2, 3, 5, 7, 11, and 13. Their sum is 41.

The answer is (D).

Question 2.

The answer is (B).

Explanation for Question 2:

The two smallest primes are 2 and 3. Their product is 6. The answer is (B).

Question 3.

The answer is (C).

Explanation for Question 3:

To be a prime number, the only factors can be 1 and the number itself: 51 has factors of 1, 3, 17, and 51; 49 has factors of 1, 7, and 49; 47 has factors of only 1 and 47. You need to try dividing 47 only by 2, 3, and 5 because 5 is the prime number closest to the square root of 47. The answer is (C).

Question 4.

The answer is (B).

Explanation for Question 4:

$$48 = 6 \times 8 = 2 \times 3 \times 2 \times 4 = 2 \times 3 \times 2 \times 2 \times 2$$

This is one way to get the prime factorization.

The answer is (B).

Question 5.

The answer is (B).

Explanation for Question 5:

The number 81 has factors of 1, 3, 9, 27, and 81.

The number 83 has factors of only 1 and 83.

You need to try dividing 83 only by 2, 3, 5, and 7.

The answer is (B).

Question 6.

The answer is (E).

Explanation for Question 6:

The only prime number squared is 2. The answer is (E).

Studyguide for the SAT Skill Quiz A: Prime Numbers

Question 7.

The answer is (B).

Explanation for Question 7:

The list of numbers in Q is 5, 8, 12, 18, ... The smallest and only prime number in Q is 5 (why are all the rest even?). The answer is (B).

Question 8.

The answer is (E).

Explanation for Question 8:

$2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 = 129$. The answer is (E).

Question 9.

The answer is (D).

Explanation for Question 9:

The numbers 37, 43, and 47 are all primes. The largest one less than 50 is 47. The answer is (D).

Question 10.

The answer is (B).

Explanation for Question 10:

The two smallest prime numbers are 2 and 3. Their sum is 5. The answer is (B).



Question 1

Which of the following is the sum of two different prime numbers?

- (A) 2
- (B) 3
- (C) 4
- (D) 6
- (E) 8

Question 2

What is the number of prime numbers between 1 and 10?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

Question 3

Which of the following is not a prime number?

- (A) 2
- (B) 3
- (C) 11
- (D) 17
- (E) 21

Question 4

What is the prime factorization of 300?

- (A) $2 \times 2 \times 3 \times 5 \times 5$
- (B) $3 \times 4 \times 25$
- (C) $2 \times 3 \times 5 \times 5$
- (D) $2 \times 3 \times 3 \times 5 \times 5$
- (E) 12×25

Question 5

If k is a prime number, then $k - 1$ could be which of the following numbers?

- (A) 4
- (B) 5
- (C) 9
- (D) 11
- (E) 14

Question 6

If r is a prime number greater than 3, which of the following is not a factor of $8r$?

- (A) $2r$
- (B) $4r$
- (C) r
- (D) $3r$
- (E) 8

Question 7

If 41 is the sum of three consecutive prime numbers, then what is the largest prime number?

- (A) 7
- (B) 11
- (C) 13
- (D) 17
- (E) 19

Question 8

Which of the following is the cube of a prime number?

- (A) 125
- (B) 64
- (C) 25
- (D) 16
- (E) 4

Question 9

What is the number of prime numbers between 1 and 30?

- (A) 6
- (B) 10
- (C) 12
- (D) 15
- (E) 29

Question 10

The set P consists of all numbers which are the product of three consecutive prime numbers. For example, the number 1,001 is in P, because $7 \times 11 \times 13 = 1,001$. What is the smallest number in P?

- (A) 0
- (B) 6
- (C) 30
- (D) 48
- (E) 105

Studyguide for the SAT Skill Quiz B: Prime Numbers



Hint for Question 1:

Write out the list of the prime numbers and find two that work.

Hint for Question 2:

Write the list and count them.

Hint for Question 3:

Find the number that has factors other than 1 and itself.

Hint for Question 4:

Keep factoring 300 until you have only prime factors.

Hint for Question 5:

Write out the list of prime numbers; then subtract 1 from each number in the list.

Hint for Question 6:

What are the factors of $8r$?

Hint for Question 7:

Write out the list of prime numbers.

Which three consecutive prime numbers do you add to get 41?

Hint for Question 8:

Which cube root is a prime number? Or write out the list of prime numbers and cube each of them.

Hint for Question 9:

Write out the list and count them.

Hint for Question 10:

Write out the list and multiply three consecutive numbers.

**Question 1.**

The answer is (E).

Explanation for Question 1:

The prime numbers are 2, 3, 5, 7, 11, 13, ... Because $8 = 3 + 5$, it is the answer.

The answer is (E).

Question 2.

The answer is (E).

Explanation for Question 2:

The prime numbers less than 10 are 2, 3, 5, and 7. There are four of them.

The answer is (E).

Question 3.

The answer is (E).

Explanation for Question 3:

The number 21 has factors of 7 and 3.

The answer is (E).

Question 4.

The answer is (A).

Explanation for Question 4:

$300 = 6 \times 50 = 2 \times 3 \times 5 \times 10 = 2 \times 3 \times 5 \times 2 \times 5$. The answer is (A).

Question 5.

The answer is (A).

Explanation for Question 5:

The number 4 is 1 less than the prime number 5. The answer is (A). (You could also just add 1 to each answer choice and see which one is prime.)

Question 6.

The answer is (D).

Explanation for Question 6:

The factors of $8r$ are 1, 2, 4, 8, r , $2r$, $4r$, $8r$. $3r$ is not one of them.

Another way to answer this question is to check divisibility. The only answer choice $8r$ is not divisible by is $3r$. The answer is (D).

Question 7.

The answer is (D).

Explanation for Question 7:

The three prime numbers you add to get 41 are 11, 13, and 17. The answer is (D).

Question 8.

The answer is (A).

Explanation for Question 8:

The cube root of 125 is the prime number 5. The answer is (A).

Question 9.

The answer is (B).

Explanation for Question 9:

The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. There are 10.
The answer is (B).

Question 10.

The answer is (C).

Explanation for Question 10:

The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, ... The smallest number in the list of products will be the product of the first three prime numbers.
 $2 \times 3 \times 5 = 30$. The answer is (C).

Math Lesson #3

Percents

Triumph College Admissions



Skill Lesson

Quiz Time!

A B C ☒ E



Hints!

A B C ☒ E



Answers!

A B C ☒ E



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PERCENTS

Percent means "hundredths" or "number divided by 100." To convert a number to a percent, multiply the number by 100.

A percent is found by dividing the part by the total and multiplying by 100.

$$\text{percent} = \frac{\text{part}}{\text{total}} \times 100$$

Example 1:

If Tanisha missed 12 out of 80 questions on a test, what percent of the questions did she miss?

Solution:

$$\text{percent} = \frac{\text{part wrong}}{\text{total}} \times 100 = \frac{12}{80} \times 100 = 0.15 \times 100 = 15\%$$

The phrase "x is n percent of y" can be written mathematically as

$$x = \frac{n}{100} \times y. \text{ The word "is" means equal (=) and the word "of"}$$

means multiply (\times). Before you multiply you must change a percent to a decimal or fraction.

$$5 \text{ is } 20\% \text{ of } 25 \text{ means } 5 = 0.20 \times 25$$

To change a fraction to a percent: first change the fraction to a decimal and then multiply by 100. (Move the decimal point **2** places to the right.)

Example 2:

Change $1/5$ to a percent.

Solution:

Change $1/5$ to the decimal 0.2 and move the decimal 2 places to the right or take its decimal value 0.2 and multiply by 100. $1/5 \times 100 = 20\%$

Studyguide for the SAT Skill Review: Percents

Common Percents to Memorize

$$1/100 = 0.01 = 1\%$$

$$1/10 = 0.1 = 10\%$$

$$1/7 = 0.1428571 = 14.3\%$$

$$1/6 = 0.1666... = 16.6\% \text{ or } 16.7\%$$

(These are truncated or rounded off to fit in an answer grid.)

$$1/5 = 0.2 = 20\%$$

$$1/4 = 0.25 = 25\%$$

$$1/3 = 0.33 = 33.3\%$$

$$1/2 = 0.5 = 50\%$$

$$3/5 = 0.6 = 60\%$$

$$1 = 1.0 = 100\%$$

$$3/2 = 1.5 = 150\%$$

Note: When you get over 1, the percents are greater than 100%.

Example 3:

What is 20% of 50?

Example 4:

5 is what percent of 2?

Solution:

Rewrite this as an algebraic equation. Let x be the unknown number.

$x = 0.20 \times 50$. Change the percent to a decimal and remember that "of" means multiply. $x = 10$

Solution:

Rewrite this as an algebraic equation: $5 = n \times 2$

Solve for n and remember

to change the answer to a percent.

$$n = \frac{5}{2} = 2.5 = 250\%$$

Example 5:

If Bill saved 20% of the normal \$40.00 price when he bought a jacket, how much money did he save?

When answering a percent word problem be sure you read the problem carefully and answer only what the question asks.

Solution:

"Of" means multiply. $20\% \text{ of } 40 = 0.20 \times 40 = 8$. Bill saved \$8.00.



Question 1

If x is a positive number, then 40% of $10x$ equals

- (A) $2x$
- (B) $4x$
- (C) $6x$
- (D) $20x$
- (E) $40x$

Question 2

What is 0.4% of 1,200?

- (A) 480
- (B) 300
- (C) 48
- (D) 4.8
- (E) 0.48

Question 3

72 is 9% of what number?

- (A) 800
- (B) 648
- (C) 480
- (D) 80
- (E) 8

Question 4

A waiter in a restaurant received a tip of \$3.75 from a customer. The customer's bill for the meal was \$25.00. What percentage of the bill was the tip?

- (A) $33\frac{3}{4}\%$
- (B) 20%
- (C) $17\frac{1}{4}\%$
- (D) 15%
- (E) $6\frac{2}{3}\%$

Question 5

A class of 50 girls and 60 boys sponsored a basketball game. If 52% of the girls and 30% of the boys attended the game, what percentage of the class went to the game?

- (A) 55%
- (B) 44%
- (C) 41%
- (D) 40%
- (E) 35%

Question 6

In 1990, the price of milk was 70% of the price in 1999. In 1999, milk was \$2.10 a gallon. What was the cost in 1990?

- (A) \$0.14
- (B) \$0.63
- (C) \$1.40
- (D) \$1.47
- (E) \$1.80

Studyguide for the SAT Skill Quiz A: Percents

Question 7

The regular price of a ticket at the Herberger Theatre is \$8.50. The student discount is 20% off the regular price. Find the cost of a student ticket.

- (A) \$1.70
- (B) \$4.25
- (C) \$6.80
- (D) \$8.30
- (E) \$10.20

Question 8

What percent of 5 is 6?

- (A) 80%
- (B) 83%
- (C) 120%
- (D) 125%
- (E) 150%

Grid-in your answer here:

Question 9

During a week in December the Dow Jones Stock Market Average rose from 9,800 to 10,878. By what percentage did the Dow Jones Stock Market increase?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Grid-in your answer here:

Question 10

Lynn bought a coat on sale that was 20% off the original price. She had a coupon that got her 10% off of the sale price. What percentage did she get off the original cost of the coat?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Studyguide for the SAT Skill Quiz A: Percents



Hint for Question 1:

What does "of" mean?

Hint for Question 2:

What does "of" mean? Rewrite the question as an equation. Be sure to change 0.4% to a decimal.

Hint for Question 3:

Rewrite the question as an equation. Be sure to change 9% to a decimal.

Hint for Question 4:

Rewrite the question as an equation. The tip is what percent of the bill?

Hint for Question 5:

This is a multi-step problem. Be sure you answer the final question. You first need to know how many went to the game. What percents do you use to find the number that attended the game? To find the percent

evaluate the fraction $\frac{\text{number who attended}}{\text{total number of students}} \times 100$.

Hint for Question 6:

Your equation comes from the first sentence.

Hint for Question 7:

You need to find the cost of the student ticket, not the discount.

Hint for Question 8:

Rewrite the question as an equation.

Hint for Question 9:

You first have to find the increase.

The percent is the $\frac{\text{increase}}{\text{original amount}} \times 100$.

Hint for Question 10:

Choose an amount for the original cost. 100 is a good choice for an original amount because it can help you easily change other numbers to a percent. For example if her final cost was \$60, she saved 40%. (The amount saved is original cost - final cost.) Figure her final cost and use it to find the percent off the original cost.



Question 1.

The answer is (B).

Explanation for Question 1:

"Of" means multiply. Change 40% to a decimal and multiply. $0.40 \times 10x = 4x$.
The answer is (B).

Question 2.

The answer is (D).

Explanation for Question 2:

"Of" means multiply. The question can be rewritten as $n = 0.004 \times 1,200$. The answer is (D).

Question 3.

The answer is (A).

Explanation for Question 3:

The question can be rewritten as $72 = 0.09 \times n$.

Solving for n you get $n = \frac{72}{0.09} = 800$. The answer is (A).

Question 4.

The answer is (D).

Explanation for Question 4:

The question can be rewritten as $n \times 25 = 3.75$.

When solving for n you get $n = \frac{3.75}{25} = 0.15$.

Changing the 0.15 to a percent equals 15%. The answer is (D).

Question 5.

The answer is (D).

Explanation for Question 5:

You first need to find out how many students attended the game. The number of girls who attended the game was $0.52 \times 50 = 26$ and the number of boys who attended the game was $0.30 \times 60 = 18$. The total attendance was $26 + 18 = 44$. This is **not** the answer. The total number of students in the class is $50 + 60 = 110$.

The percent of the class that attended is $\frac{44}{110} = 0.40 = 40\%$. The answer is (D).

Studyguide for the SAT Skill Quiz A: Percents

Question 6.

The answer is (D).

Explanation for Question 6:

The equation is "milk was 70% of the price" or $m = 0.70 \times 2.10 = 1.47$. The answer is (D).

Question 7.

The answer is (C).

Explanation for Question 7:

One way to solve this problem follows. If the discount is 20% off the regular price then the current cost is 80% of the regular price. Take 80% of 8.50 to find the answer directly \$6.80. The answer is (C). A longer way is to find the discount (\$1.70) and subtract it from the regular cost (\$8.50) to get \$6.80.

Question 8.

The answer is (C).

Explanation for Question 8:

The equation is $n \times 5 = 6$. Solving for n you get $n = \frac{6}{5} = 1.2 = 120\%$. The answer is (C).

Question 9.

The answer is 11.

Explanation for Question 9:

The increase is $10,878 - 9,800 = 1,078$.

This part gets divided by the original (beginning) amount.

The percent = $\frac{1,078}{9,800} = 0.11 = 11\%$. The answer is 11.

Question 10.

The answer is 28.

Explanation for Question 10:

Some problems can be solved by choosing a number for the original cost. A good number to choose is one that would make the problem easy. Because you're looking for a percent as an answer, a good original number for this problem would be 100. The sale price is then $80\% \times \$100$ or \$80. (Don't forget a 20% discount means the item costs 80% of the original price.) The coupon gets Lynn another 10% off the SALE price. Her final cost is $90\% \times 80$ or \$72. (Again, a 10% savings means the item costs 90% of the SALE price.) The question asks what percent she got off the original cost of the coat. The original cost was \$100 and because she is paying \$72 for the coat, she has \$28 off.

Her percent off is $\frac{\text{part off}}{\text{total}} \times 100 = \frac{28}{100} \times 100 = 28\%$.

Do you see why 100 was a good number for the original cost?

The answer is 28.



Question 1

An auctioneer sold goods amounting to \$325,000. If the auctioneer kept 7% of the total for his fee, how much money did the auctioneer earn?

- (A) \$2,275
- (B) \$22,750
- (C) \$35,125
- (D) \$227,500
- (E) \$302,250

Question 2

A salesman is paid a yearly salary of \$35,000 plus a commission of 5% on total sales of \$204,000. Compute the salesman's total earnings for the year.

- (A) \$10,200
- (B) \$45,200
- (C) \$70,000
- (D) \$75,200
- (E) \$97,000

Question 3

A can of tennis balls regularly sells for \$3.95. They are on sale at 40% off. What is the amount of the discount?

- (A) \$0.16
- (B) \$1.58
- (C) \$2.37
- (D) \$3.55
- (E) \$5.53

Question 4

Last month, the Johnson family paid \$120 for heating. By lowering the thermostat setting, they reduced this month's bill to \$105. Find the percent of decrease compared to last month.

- (A) 5.5%
- (B) 12.5%
- (C) 15%
- (D) 85%
- (E) 87.5%

Question 5

The price of sofas at a store is \$500. Display models are sold at 64% off. What is the price of a display model?

- (A) \$120
- (B) \$180
- (C) \$320
- (D) \$436
- (E) \$820

Question 6

In 1992, the value of a painting was \$6,000. In 1996, the value was \$7,500. What was the percent of increase in value?

- (A) 15%
- (B) 20%
- (C) 25%
- (D) 125%
- (E) 150%

Question 7

During the women's NCAA Basketball Tournament, Jane made 13 successful free throws. This was 65% of her attempted free throws. How many free throws did Jane attempt?

- (A) 10
- (B) 12
- (C) 18
- (D) 20
- (E) 25

Question 8

Suppose a piece of fruit provides 90 units of vitamin C. If this is 150% of the total recommended daily allowance for vitamin C, find the total recommended daily allowance, to the nearest unit.

- (A) 37 units
- (B) 45 units
- (C) 60 units
- (D) 135 units
- (E) 240 units

Question 9

Nancy spent 6 vacation days at the beach. This was 30% of her total number of vacation days. How many days was her vacation?

- (A) 12
- (B) 20
- (C) 24
- (D) 36
- (E) 42

Question 10

Jane earned \$30.00 on Monday. This was 25% of her weekly earnings. How much does she earn per week?

- (A) \$150
- (B) \$120
- (C) \$75
- (D) \$55
- (E) \$25

Studyguide for the SAT Skill Quiz B: Percents



Hint for Question 1:

Your equation is in the second sentence.

Hint for Question 2:

His yearly salary will be \$35,000 plus his commission. How do you find his commission?

Hint for Question 3:

What numbers do you multiply?

Hint for Question 4:

Be sure to find the decrease and remember $\frac{\text{part}}{\text{total}} \times 100$.

Hint for Question 5:

If display models are 64% off, their cost is what percent of non-display models.

Hint for Question 6:

First find the increase.

Hint for Question 7:

This is a tougher problem. One way to solve is to use backsolving.

Hint for Question 8:

One way to solve is to use backsolving. Don't forget what 150% is as a decimal.

Hint for Question 9:

One way to solve is to use backsolving.

Hint for Question 10:

One way to solve is to use backsolving.



Question 1.

The answer is (B).

Explanation for Question 1:

Your equation is $0.07 \times 325,000 = \text{fee}$. Solving for fee you get $\text{fee} = 22,750$.

The answer is (B).

Question 2.

The answer is (B).

Explanation for Question 2:

His yearly salary will be $\$35,000 + 0.05 \times \$204,000 = \$35,000 + \$10,200 = \$45,200$. The answer is (B).

Question 3.

The answer is (B).

Explanation for Question 3:

The discount is 40% of the original price. The discount equals $.040 \times 3.95 = 1.58$.

The answer is (B).

Question 4.

The answer is (B).

Explanation for Question 4:

The decrease is \$15. The percent of decrease is $\frac{15}{120} \times 100 = 12.5\%$.

You use 120 because it was the original or beginning cost.

The answer is (B).

Question 5.

The answer is (B).

Explanation for Question 5:

Display models cost 36% (100 - 64) of non-display models.

The cost of a display model is $0.36 \times 500 = 180$. A longer

way is to find the discount $0.64 \times 500 = 320$ and subtract

320 from 500 to get 180. The answer is (B).

Question 6.

The answer is (C).

Explanation for Question 6:

The increase is 1,500. The percent increase is then $\frac{1,500}{6,000} \times 100 = 25\%$.

Again, be sure to divide by the original or beginning cost.

The answer is (C). (You didn't guess choice (B) did you?)

Question 7.

The answer is (D).

Explanation for Question 7:

backsolving involves substituting in the answer choices until

you get 65%. Start with choice (C). $\frac{13}{18} = 72.2\%$. This

value is too big. Because we want a smaller percent we need

a larger denominator. Try choice (D) next. $\frac{13}{20} = 65\%$. This is

our answer. You also could get the answer by solving the equation $0.65 \times F = 13$ where F = number of free throws.

Solving for F you get $F = \frac{13}{0.65} = 20$. The answer is (D).

Question 8.

The answer is (C).

Explanation for Question 8:

Again take each answer choice and multiply by $150\% = 1.5$ until you get 90. Start with choice (C). $60 \times 1.5 = 90$. This is our answer.

You could also get the answer by solving the equation $1.5 \times V = 90$ where V = daily allowance for vitamin C. Solving for V you get

$V = \frac{90}{1.5} = 60$. The answer is (C).

Question 9.

The answer is (B).

Explanation for Question 9:

Take each answer choice and multiply by $30\% = 0.30$ until you get 6. Start with choice (C). $24 \times 0.30 = 7.2$. This is too big so you need to use a smaller number. Choose (B) as your next attempt. $20 \times 0.30 = 6$. This is your answer. You also could get the answer by solving the equation $0.30 \times D = 6$ where D is the number

of vacation days. Solving for D you get $D = \frac{6}{0.30} = 20$.
The answer is (B).

Question 10.

The answer is (B).

Explanation for Question 10:

Are you tired of these problems yet? Multiply each answer choice by $25\% = 0.25$ until you get 30. Always start with choice (C). $0.25 \times 75 = 18.75$. This is too small so you need to try a bigger number. Choose (B) as your next attempt. $0.25 \times 120 = 30$. This is our answer. The answer is (B). There are other ways to solve this problem. Look at the solution to problem number 9 for a hint on another approach.

Math Lesson #4

Number Sequences

Triumph College Admissions



Skill Lesson

Quiz Time!

A B C ☒ E



Hints!

A B C ☒ E



Answers!

A B C ☒ E



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NUMBER SEQUENCES/COUNTING

The SAT often contains one or two questions that test whether you know how to deal with a sequence or series of numbers.

Example 1:

The series of numbers $\{1, -1, 1, -1, \dots\}$ consists of alternating 1's and -1's. What is the 176th term in the series?

Solution:

It's -1. This answer is easy if you notice that the 2nd and 4th terms are -1, and in fact **every** even-numbered term is -1.

Example 2:

The series of numbers $\{1, -1, 1, -1, \dots\}$ consists of alternating 1's and -1's. What is the SUM of the first 99 terms in the series?

Solution:

It's 1. Again, this question is easy if you notice that every even-numbered term cancels out the previous odd-numbered term:

The sum of the first 2 terms is 0.

The sum of the first 3 terms is 1.

The sum of the first 4 terms is 0 again.

The sum of the first 98 terms (or any even number) is 0; and the sum of the first 99 terms (or any odd number) is 1 again.

Hint: The SAT will never require you to add up 99 numbers, so as soon as you see a problem like this, you know there is a shortcut.

Studyguide for the SAT Skill Review: Number Sequences/Counting

Some questions on the SAT test whether you can write (or manipulate) a series of numbers expressed as variables.

Here are some tips:

- * Consecutive integers (like 10, 11, 12,...) can be written $(n, n + 1, n + 2, \dots)$ because each term is 1 greater than the previous term.
- * Consecutive even (or odd) numbers can be written $(n, n + 2, n + 4, \dots)$ because each term is 2 greater than the previous term.

Example 3:

The sum of three consecutive even numbers is 24.

What is the smallest of the three numbers?

Solution:

Given: The sum of three consecutive even numbers is 24.

You write: $n + (n + 2) + (n + 4) = 24$
because "consecutive even" means they are 2 apart.

Solve for n: $3n + 6 = 24$
 $3n = 18$
 $n = 6$

What appeared to be a hard question is not too bad once you know how to write an equation for the number sequence!

Check your answer! It's a good idea to double-check your answer. In this case, $6 + 8 + 10$ does add up to 24.

Here is another way to solve the previous question. Because there is an odd number of consecutive integers, the middle number will be the **average** of their sum. The average of the three numbers is $24/3 = 8$. This is the middle number (the second number). The consecutive even integers are 6, 8, and 10. The smallest integer is 6.

The proof for this fact follows.

Let x equal the middle integer.

$x - 2$ is the smallest integer.

$x + 2$ is the largest integer.

The sum of the three integers is $(x - 2 + x + x + 2) = 3x$. The average of this is $3x/3 = x$. The middle integer will always be the average of the sum, if the number of integers is odd.

This method is often quicker than letting some variable equal the number.

Example 4:

Let's try another question similar to the last one:

The sum of three consecutive ODD numbers is 45.

What is the MIDDLE of the three numbers?

Solution:

Given: The sum of three consecutive ODD numbers is 45.

You write: $(n - 2) + n + (n + 2) = 45$
 (Let n be the middle value we are looking for.)

Solve for n : $3n = 45$
 $n = 15$

Counting

There is a difference between

- * counting the number of integers **between** two integers, and
- * counting the number of integers **from** one integer to another integer.

For example, there are two integers **between** 2 and 5, but there are four integers **from** 2 to 5.

The SAT usually has a few math questions that test whether you know when to use each type of counting. In particular, quite a few students (incorrectly) count the number of integers between two integers in questions when they **should** count from one integer to the other integer.

If $y > x$,

- * There are $(y - x - 1)$ integers **between** x and y
- * The **difference** between y and x is $(y - x)$
- * There are $(y - x + 1)$ integers **from** x to y

For example, there are 2 integers **between** 2 and 5, the **difference** between 5 and 2 is 3, and there are 4 integers **from** 2 to 5.

Example 5:

How many Honorable Mention awards were given away if they were given to the students who placed 4th through 10th in a contest?

Incorrect Solution:

A typical incorrect solution:

Many students incorrectly answer $10 - 4 = 6$ awards. Wrong!

Correct Solution:

The correct answer is 7. There are $10 - 4 + 1$ awards given away if they are given to students who placed 4th through 10th (inclusive).



Question 1

If customers numbered 212 through 304 were served lunch between 12:30 and 1:30, how many customers were served during that time?

- (A) 91
- (B) 92
- (C) 93
- (D) 192
- (E) 193

Question 2

In the last hour on Election Day, voters 51,312 to 55,126 voted. How many voters voted during the last hour?

- (A) 3,813
- (B) 3,814
- (C) 3,815
- (D) 3,816
- (E) 3,817

Question 3

Joan begins reading at the top of page 15 and finishes at the bottom of page x . If the pages are numbered and read consecutively, and if there are no blank pages, how many pages did she read?

- (A) 15
- (B) 16
- (C) $x - 15$
- (D) $x - 14$
- (E) $x - 13$

Question 4

Peter washes cars on weekends by himself. On Saturday, he washed cars 1 through R . On Sunday, he washed cars $R + 1$ through S . If the cars are numbered and washed consecutively, how many cars did he wash on Sunday?

- (A) $R - S$
- (B) $S - R$
- (C) $S - R - 1$
- (D) $R - S + 1$
- (E) $S - R + 1$

Question 5

If a straight fence has 406 fence posts, how many fence posts are between the 50th post and the 406th post?

- (A) 354
- (B) 355
- (C) 356
- (D) 357
- (E) 358

Question 6

The sum of five consecutive integers is 120. What is the largest of the five integers?

- (A) 23
- (B) 24
- (C) 25
- (D) 26
- (E) 27

Question 7

The sum of two integers is 33. The difference between the two integers is 3. What is the smaller of the two integers?

- (A) 11
- (B) 15
- (C) 18
- (D) 32
- (E) 36

Question 8

The series $\{-2, 1, 1, -2, 1, 1, \dots\}$ consists of the alternating terms -2 , 1 , and 1 , as shown. What is the sum of the first 34 terms in the series?

- (A) -2
- (B) 0
- (C) 1
- (D) 2
- (E) 4

Question 9

The series $\{-2, 3, -2, 3, \dots\}$ consists of the alternating terms -2 and 3 , as shown. What is the sum of the first 11 terms in the series?

- (A) -2
- (B) 1
- (C) 2
- (D) 3
- (E) 11

Question 10

In a series of integers, each term (after the first term) is twice as large as the prior term. If the first term is 3 , what is the third term?

- (A) 3
- (B) 6
- (C) 9
- (D) 12
- (E) 27



Hint for Question 1:

Subtracting the two numbers does not include customer 212 in the count, so 1 more must be added.

Hint for Question 2:

Subtract the two numbers and add 1.

Hint for Question 3:

Subtract 15 from x and add 1.

Hint for Question 4:

Subtract $R + 1$ from S and add 1 to include car $R + 1$.

Hint for Question 5:

Subtract 50 from 406 and subtract 1 more to exclude post 406.

Hint for Question 6:

One way to solve is to use backsolving. Select an answer choice, make it the largest integer, write the other four and add them together. The choice you use to get 120 is the answer.

Hint for Question 7:

Write two equations and solve for the smallest integer.
The equations are $x + y = 33$ and $x - y = 3$.

Hint for Question 8:

The series is grouped in threes, with a sum of zero. How many groups are in 34?
What number(s) remain?

Hint for Question 9:

How many groups of 2 are in 11 terms? What is the sum of each group?
What number remains?

Hint for Question 10:

Write the first three terms of the series.



Question 1.

The answer is (C).

Explanation for Question 1:

$304 - 212 + 1 = 93$. The answer is (C).

Question 2.

The answer is (C).

Explanation for Question 2:

$55,126 - 51,312 + 1 = 3,815$. The answer is (C).

Question 3.

The answer is (D).

Explanation for Question 3:

$x - 15 + 1 = x - 14$. The answer is (D).

Question 4.

The answer is (B).

Explanation for Question 4:

$S - (R + 1) + 1 = S - R - 1 + 1 = S - R$. The answer is (B).

Question 5.

The answer is (B).

Explanation for Question 5:

$406 - 50 - 1 = 355$. The answer is (B). The word "between" means you exclude the end numbers.

Question 6.

The answer is (D).

Explanation for Question 6:

One way to solve is to select an answer choice and see if it solves the problem.

Choice	Previous 4 numbers	Sum	= to 120?
25	24, 23, 22, 21	115	No. Too small
26	25, 24, 23, 22	120	Yes. The answer is (D).

There are other ways to solve this problem.

*** Let x be the largest number.

$x - 1$ the next smaller

$x - 2$ the second smaller

$x - 3$ the third smaller

$x - 4$ the fourth smaller

$$x + x - 1 + x - 2 + x - 3 + x - 4 = 120$$

$$5x - 10 = 120$$

$$5x = 130$$

$$x = 26.$$

*** Another way. Because there is an odd number of consecutive numbers, the middle number will be the average of their sum. The average of the five numbers is $120/5 = 24$. This is the middle number (the third number). The consecutive numbers are 22, 23, 24, 25, and 26. The largest is 26.

Question 7.

The answer is (B).

Explanation for Question 7:

$$x + y = 33$$

$$+ \underline{x - y = 3}$$

$$2x = 36$$

$$x = 18$$

$$18 + y = 33$$

$y = 15$. The smallest number is 15. The answer is (B).

Question 8.

The answer is (A).

Explanation for Question 8:

Divide 3 into 34 to find the number of groups of 3 in the first 34 terms. $34/3 = 11$ plus a remainder of 1. This means -2, 1, 1 occurs 11 times with one more term left.

The term left is -2. The answer will be $(-2 + 1 + 1) \times 11 + -2 = 0 \times 11$

$+ -2 = -2$. The answer is (A).

Question 9.

The answer is (D).

Explanation for Question 9:

Divide 2 into 11 to find the number of groups of 2 in the first 11 terms. $11/2 = 5$ plus a remainder of 1. This means the group -2, 3 occurs 5 times with one more term left.

The term left is -2. The answer will be $(-2 + 3) \times 5 + -2 = 1 \times 5 + -2 = 3$.

The answer is (D).

Question 10.

The answer is (D).

Explanation for Question 10:

Write the first three terms of the series. 3, 6, 12. The third term is 12.

The answer is (D).



Question 1

3, 4, 6, 9, 13, ...

What is the next number in the sequence above?

- (A) 14
- (B) 15
- (C) 16
- (D) 17
- (E) 18

Question 2

6, 10, 14, 18, ...

In the sequence above, each term after the first is 4 greater than the preceding term. Which of the following is not a term in the sequence?

- (A) 86
- (B) 94
- (C) 146
- (D) 168
- (E) 210

Question 3

A deck of cards is dealt one card at a time to 5 players. Player 1 gets the first card, player 2 gets the second card, and so forth, until each player has one card. The next card is dealt to player 1 and the pattern continues.

Which player receives the 28th card?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

Question 4

-1, 1, -1, 1, -1, 1, ...

In the above sequence, what is the sum of the first 501 numbers?

- (A) -2
- (B) -1
- (C) 0
- (D) 1
- (E) 2

Question 5

2, 5, 8, 11, 14, ...

In the sequence above, what is the 200th term?

- (A) 605
- (B) 602
- (C) 600
- (D) 599
- (E) 596

Question 6

The sum of three consecutive odd integers is 81. What is the smallest integer of the three?

- (A) 25
- (B) 26
- (C) 27
- (D) 29
- (E) 31

Question 7

For each customer served, Ricki gets paid a \$0.25 commission. Last Friday, Ricki served customer numbered 35 through 60. How much was her Friday commission?

- (A) \$27.00
- (B) \$26.00
- (C) \$25.00
- (D) \$6.50
- (E) \$6.25

Question 8

In a theme park, the most popular roller coaster has seats numbered 1 through 26. If each seat holds 2 people, what is the total capacity of the roller coaster?

- (A) 25
- (B) 26
- (C) 50
- (D) 52
- (E) 54

Question 9

What is the unit digit of 3^{86} ?

- (A) 0
- (B) 1
- (C) 3
- (D) 7
- (E) 9

Question 10

1, -1, 1, 1, -1, 1, 1, -1, 1, ...

What is the sum of the first 30 terms of the above sequence?

- (A) 30
- (B) 10
- (C) 9
- (D) 8
- (E) 0



Hint for Question 1:

Look for a pattern. What is the difference between each term of the sequence? Does the difference remain a constant or does it change? If it changes is there a pattern in the change? It may be easier if you make a table.

Number	Difference
3	
	1
4	
	2
6	
	3
9	
	4
13	Can you find the pattern?

Hint for Question 2:

Can you find a pattern? What happens if you divide each number in the sequence by 4? Look at the remainder.

Hint for Question 3:

How many groups of 5 are in 28?

Hint for Question 4:

What is the sum of each pair of numbers? How many groups of 2 are in 501? What number remains?

Hint for Question 5:

What is the pattern? What number is added to each term? Can you develop a relationship between the number you add to each term and each term of the sequence?

Hint for Question 6:

You could use backsolving.

Hint for Question 7:

How many customers did Ricki serve? Multiply the number of customers by \$0.25.

Hint for Question 8:

Multiply the number of seats by 2.

Hint for Question 9:

Write the sequence of unit digits. Begin with $3^1, 3^2, 3^3, 3^4, 3^5, \dots$
Look for a pattern in the unit digits.

Hint for Question 10:

Be careful. The repeat group of the sequence is 1, -1, 1.
The sum of this repeat group is 1.
How many groups of 3 are in the first 30 terms?



Question 1.

The answer is (E).

Explanation for Question 1:

The next term in the sequence is 5 more than the last term. $13 + 5 = 18$

The answer is (E).

Question 2.

The answer is (D).

Explanation for Question 2:

One way to answer the problem is to divide each number of the given sequence by 4.

$$6/4 = 1.5$$

$$10/4 = 2.5$$

$$14/4 = 3.5$$

$$18/4 = 4.5$$

Each answer has a .5.

$$86/4 = 21.5 \quad \text{Ok}$$

$$94/4 = 23.5 \quad \text{Ok}$$

$$146/4 = 36.5 \quad \text{Ok}$$

$$168/4 = 42 \quad \text{Oops. No .5. This is not in the sequence.}$$

The answer is (D).

Other ways would include the following:

Because you add 4 to each preceding term, any term in the sequence will be a multiple of 4 **after** you subtract the first term, 6.

$$(10 - 6)/4 = 1$$

$$(14 - 6)/4 = 2$$

$$(18 - 6)/4 = 3$$

Use this information to answer the question.

Question 3.

The answer is (C).

Explanation for Question 3:

There are 5 to a group in this problem. $28/5 = 5$ with a remainder of 3.

You will have 5 complete rounds with 3 more cards dealt.

Player 3 will get the 28th card. The answer is (C).

Question 4.

The answer is (B).

Explanation for Question 4:

Divide 2 into 501 to find the number of groups of 2 in the first 501 terms. $501/2 = 250$ plus a remainder of 1. This means -1, 1 occurs 250 times with one more term left. The term left is -1. The answer will be the sum of $(-1 + 1) \times 250$ and $-1 = 0 \times 250 + -1 = -1$. The answer is (B).

Question 5.

The answer is (D).

Explanation for Question 5:

Look for a pattern

Number	Difference
2	
	3
5	
	3
8	
	3
11	
	3
14	

The difference between each number is 3. How can we get a relationship between the 3 and a number in the sequence?

Look at the following table.

Term	Number in Sequence	Relation to 3
1	2	$3 - 1$
2	5	$2(3) - 1$
3	8	$3(3) - 1$
4	11	$4(3) - 1$

Wow, do you see the importance of finding the difference? You can find any term in the sequence by multiplying its position by 3 and subtracting 1.

Term	Number in Sequence	Relation to 3
200	?	$200(3) - 1 = 600 - 1 = 599$

The answer is (D).

Question 6.

The answer is (A).

Explanation for Question 6:

Use backsolving. Make a table.

Answer Choice	Other 2 numbers	Sum	Sum = 81
27	29, 31	87	No. Too large
26	No. Not odd.		
25	27, 29	81	Yes.

The answer is (A).

Here is the quickest way to solve this problem. $81/3 = 27$. (Remember, find the average to get the middle number, if the number of terms is odd.) The number 27 is the middle integer; 25, 27, and 29 are the three integers. The number 25 is the smallest.

Question 7.

The answer is (D).

Explanation for Question 7:

The number of customers is $60 - 35 + 1 = 26$. The commission will be $26 \times \$0.25 = \6.50 . The answer is (D).

Question 8.

The answer is (D).

Explanation for Question 8:

There are 26 seats on the roller coaster. You really don't have to subtract and add 1. The capacity is $26 \times 2 = 52$. The answer is (D).

Question 9.

The answer is (E).

Explanation for Question 9:

Write a sequence of unit digits.

$$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, \text{ etc.}$$

You need to multiply only the last digit by 3 to get the next unit digit.

The pattern 3, 9, 7, 1 keeps repeating.

The sequence of unit digits is 3, 9, 7, 1, 3, 9, 7, 1, 3, ...

Note that the sequence repeats every 4 digits.

To answer the question, how many groups of 4 are in 86?

$86/4 = 21$ with a remainder of 2. This means there are 21 groups of 4 plus 2 more numbers. The two numbers will be 3 and 9.

The unit digit for 3^{86} will be 9.

The answer is (E).

Question 10.

The answer is (B).

Explanation for Question 10:

The repeating digits are 1, -1, 1. The sum of these digits is 1.

Divide 3 into 30 to find the number of groups of 3 in the first 30 terms.

$30/3 = 10$ with no remainder. This means the group 1, -1, 1 occurs 10 times.

The answer will be $(1 + -1 + 1) \times 10 = 1 \times 10 = 10$. The answer is (B).



Question 1

If customers numbered 273 through 408 were served lunch between 12:30 and 1:30, how many customers were served during that time?

- (A) 135
- (B) 136
- (C) 137
- (D) 235
- (E) 236

Question 2

Joan begins reading at the top of page 35 and finishes at the bottom of page p . If the pages are numbered and read consecutively, and if there are no blank pages, how many pages did she read?

- (A) 35
- (B) 36
- (C) $p - 35$
- (D) $p - 34$
- (E) $p - 33$

Question 3

If a straight fence has 327 fence posts, how many fence posts are between the 38th post and the 327th post?

- (A) 290
- (B) 289
- (C) 288
- (D) 287
- (E) 286

Question 4

The sum of five consecutive integers is 640. What is the largest of the five integers?

- (A) 126
- (B) 127
- (C) 128
- (D) 129
- (E) 130

Question 5

The series $\{-2, 2, 1, -2, 2, 1, \dots\}$ consists of the alternating terms -2 , 2 , and 1 , as shown. What is the sum of the first 34 terms in the series?

- (A) 1
- (B) 9
- (C) 11
- (D) 13
- (E) 34

Question 6

3, 4, 8, 17, 33, ...

What is the next number in the sequence above?

- (A) 34
- (B) 39
- (C) 50
- (D) 58
- (E) 68

Question 7

A deck of cards is dealt one card at a time to 5 players. Player 1 gets the first card, player 2 gets the second card, and so forth, until each player has one card. The next card is dealt to player 1 and the pattern continues.

Which player receives the 20th card?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

Question 8

What is the unit digit of 3^{43} ?

- (A) 0
- (B) 1
- (C) 3
- (D) 7
- (E) 9

Question 9

1, 3, 5, 7, 9, ...

In the sequence above, what is the 200th term?

- (A) 397
- (B) 398
- (C) 399
- (D) 400
- (E) 401

Question 10

1, -1, 1, -1, 1, -1, 1, ...

What is the sum of the first 30 terms of the above sequence?

- (A) 30
- (B) 10
- (C) 9
- (D) 8
- (E) 0



Hint for Question 1:

Subtracting the two numbers does not include customer 273 in the count, so 1 more must be added.

Hint for Question 2:

Subtract 35 from p and add 1.

Hint for Question 3:

Subtract 38 from 327 and subtract 1 more to exclude post 327.

Hint for Question 4:

One way to solve is to use backsolving. Select an answer choice, make it the largest integer, write the other 4, and add them together. The choice you use to get, 640, is the answer.

Hint for Question 5:

The series is grouped in threes, with a sum of one. How many groups are in 34? What number(s) remain?

Hint for Question 6:

Look for a pattern. What is the difference between each term of the sequence? Does the difference remain a constant or does it change? If it changes is there a pattern in the change? It may be easier if you make a table.

Number	Difference
3	
	1
4	
	4
8	
	9
17	
	16
33	Can you find the pattern?

Hint for Question 7:

How many groups of 5 are in 20?

Hint for Question 8:

Write the sequence of unit digits. Begin with $3^1, 3^2, 3^3, 3^4, 3^5, \dots$
Look for a pattern in the unit digits.

Hint for Question 9:

What is the pattern? What number is added to each term? Can you develop a relationship between the number you add to each term and each term of the sequence?

Hint for Question 10:

Be careful. The repeat group of the sequence is 1, -1.
The sum of this repeat group is 0.
How many groups of 2 are in the first 30 terms?



Answers!



Question 1.

The answer is (B).

Explanation for Question 1:

$408 - 273 + 1 = 136$. The answer is (B).

Question 2.

The answer is (D).

Explanation for Question 2:

$p - 35 + 1 = p - 34$. The answer is (D).

Question 3.

The answer is (C).

Explanation for Question 3:

$327 - 38 - 1 = 288$. The answer is (C).

The word "between" means you exclude the end numbers.

Question 4.

The answer is (E).

Explanation for Question 4:

One way to solve is to select an answer choice and see if it solves the problem.

Answer Choice	Previous 4 numbers	Sum	= to 640?
128	127, 126, 125, 124	630	No, too small
129	128, 127, 126, 125	635	No, too small
130	129, 128, 127, 126	640	Yes.

The answer is (E).

There are other ways to solve this problem.

*** Let x be the largest number.

$x - 1$ the next smaller

$x - 2$ the second smaller

$x - 3$ the third smaller

$x - 4$ the fourth smaller

$$x + x - 1 + x - 2 + x - 3 + x - 4 = 640$$

$$5x - 10 = 640$$

$$5x = 650$$

$$x = 130$$

*** Another way. Because there is an odd number of consecutive integers, the middle number will be the average of their sum. The average of the 5 numbers is $640/5 = 128$. This is the middle number (the third number).

The consecutive numbers are 126, 127, 128, 129, and 130.

The largest is 130.

Question 5.

The answer is (B).

Explanation for Question 5:

Divide 3 into 34 to find the number of groups of 3 in the first 34 terms.

$$34/3 = 11 \text{ plus a remainder of } 1.$$

This means the group -2, 2, 1 occurs 11 times with one more term left.

The term left is -2. The answer will be the sum of $(-2 + 2 + 1) \times 11$ and $-2 = 1 \times 11 + -2 = 9$. The answer is (B).

Question 6.

The answer is (D).

Explanation for Question 6:

The next term in the sequence is 25 more than the last term. The difference is the square position of the term. $33 + 25 = 58$. The answer is (D).

Question 7.

The answer is (E).

Explanation for Question 7:

There are 5 to a group in this problem. $20/5 = 5$ with no remainder.
You will have five complete rounds. Player 5 will get the 20th card.
The answer is (E).

Question 8.

The answer is (D).

Explanation for Question 8:

Write a sequence of unit digits.

$$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, \text{ etc.}$$

You need to multiply only the last digit by 3 to get the next unit digit.

The pattern 3, 9, 7, 1 keeps repeating.

The sequence of unit digits is 3, 9, 7, 1, 3, 9, 7, 1, 3,...

Note that the sequence repeats every 4 digits.

To answer the question, how many groups of 4 are in 43? $43/4 = 10$
with a remainder of 3. This means there are 10 groups of 4 plus 3 more numbers.

The three numbers will be 3, 9, and 7. The unit digit for 3^{43} will be 7.

The answer is (D).

Question 9.

The answer is (C).

Explanation for Question 9:

Look for a pattern.

Number	Difference
1	
	2
3	
	2
5	
	2
7	
	2
9	

The difference between each number is 2. How can we get a relationship between the 2 and a number in the sequence?

Look at the following table.

Term	Number in Sequence	Relation to 3
1	1	$2 - 1$
2	3	$2(2) - 1$
3	5	$3(2) - 1$
4	7	$4(2) - 1$

Wow, do you see the importance of finding the difference? You can find any term in the sequence by multiplying its position by 2 and subtracting 1.

Term	Number in Sequence	Relation to 3
200	?	$200(2) - 1 = 400 - 1 = 399$

The answer is (C).

Question 10.

The answer is (E).

Explanation for Question 10:

The repeating digits are 1, -1. The sum of these digits is 0. Divide 2 into 30 to find the number of groups of 2 in the first 30 terms. $30/2 = 15$ with no remainder.

This means the group 1, -1 occurs 15 times.

The answer will be $(1 + -1) \times 15 = 0 \times 15 = 0$.

The answer is (E).

Math Lesson #5

Sets

Triumph College Admissions



Skill Lesson

Quiz Time!



Hints!



Answers!



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Sets

We denote a set by using braces $\{ \}$ and often label a set by using a capital letter. For example let $A = \{1, 4, 6, 9\}$.

Each individual object in the set is considered a member or element of the set. Thus 1 is an element of the above set. The symbol " \in " is read "is an element of" or "is a member of". The notation $1 \in \{1, 2, 3\}$ is read "1 is an element of the set of numbers 1, 2, and 3".

If the elements of a set are numbers, you do not have to put them in order. However, it can make set comparisons easier if you do put the numbers of a set in order.

A set is considered finite if it has a specific number of elements (the set ends). A is a finite set.

Using three dots in a set indicates that the pattern started continues. You usually want to make sure you have at least 3 representations of the pattern you wish to form. The following set uses the three dots to indicate to the reader that there is a pattern and that there are other members of the set not explicitly shown: $B = \{3, 6, 9, \dots 99\}$. You use this notation when the set may be too big to write all of the elements. By repeating the desired pattern and using the three dots it is understood that the numbers 12 and 96, along with all the other multiples of 3 up to 99, are members of set B.

A set is considered infinite if the number of elements cannot be counted. The following is an example of an infinite set: $B = \{2, 4, 6, 8, \dots\}$. Because there is no ending number this set is infinite. It has no ending.

Two sets are considered equivalent if they have the same number of elements. If set $R = \{a, b, c\}$ and set $S = \{5, 10, 15\}$, then sets R and S are equivalent.

Two sets are considered equal if they have exactly the same elements. If set $X = \{-1, 3, 4, 5\}$ and set $Y = \{3, -1, 5, 4\}$ then $X = Y$. They have exactly the same elements. Remember, the elements of a set do not have to be in order.

A set can have no elements in it. If this is the case, it is called the empty set or null set. The symbol for the empty set is either $\{ \}$ (note this is empty) or \emptyset . Never put one symbol inside the other. Here is an example of the empty set:

$\emptyset = \{x \text{ such that } x \text{ is both a positive and a negative integer}\}$

Studyguide for the SAT Skill Review: Sets

A set can be represented in three different ways.

1. As a list or roster that names the specific elements $\{1, 3, 7\}$
2. As a definition that defines the terms of the set
 $\{x \text{ such that } x \text{ is an integer between } 5 \text{ and } 500\}$
3. As a number line (the set of real numbers between -2 and 5).



Number Systems

Some SAT problems may ask students to identify specific types of numbers. Often when students are asked to name a number they respond 2 or 5 or 8 or 150. The students are correct with their responses, but they have responded with only one type of number. There are many different number systems, and you should know the names and definitions of each of them.

Here is a review of the different number systems and the letters that often represent them.

Natural numbers

$$N = \{1, 2, 3, \dots\}$$

It is important to note that the fraction $\frac{1}{2}$ is NOT in the set because it is not specifically mentioned.

Whole numbers

$$W = \{0, 1, 2, \dots\}$$

Whole numbers are almost the same set as the natural numbers. The whole number system contains the element 0.

Integers

$$I = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

Integers are the natural numbers and the opposites of the natural numbers plus the element 0.

Studyguide for the SAT Skill Review: Sets

Rational numbers

$$Q = \left\{ \frac{p}{q} \right\}, \text{ where } p \in I \text{ and } q \in N$$

Think of Q as representing quotients. Rational numbers are often referred to as the set of fractions. Remember that any integer can be expressed as a fraction. So an integer is also a rational number. As a decimal, a rational number either terminates (ends) or repeats.

Irrational numbers

$Ir = \{\text{non-repeating and non-terminating decimals}\}$

Some examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, π , 1.010010001...

(Note that the last number neither terminates, nor repeats.)

Real numbers

$$R = \{Q \text{ and } Ir\}$$

The union of two sets is a set containing all of the elements of the sets.

The symbol that indicates the union is \cup . For example let $A = \{1, 3, 5, 7, 8\}$ and $B = \{2, 5, 7, 10\}$. $A \cup B = \{1, 2, 3, 5, 7, 8, 10\}$. The union is all of the elements from the sets. Note that you only need to list an element once for it to be in the set. That is why the 5 and 7 are not repeated.

The intersection of two sets is a set containing the elements common to both sets. The symbol that indicates the intersection is \cap . For example let $A = \{1, 3, 5, 7, 8\}$ and $B = \{2, 5, 7, 10\}$. $A \cap B = \{5, 7\}$.

Here are two more things to know.

When talking about sets the word "and" indicates the intersection of two sets. For example if $A = \{1, 3, 5, 7, 8\}$, $B = \{2, 5, 7, 10\}$ and you are asked to find the set of elements belonging to sets A and B , the answer is the intersection of set A and set B . The set of elements belonging to sets A and B is $\{5, 7\}$.

When talking about sets the word "or" indicates the union of two sets. For example if $A = \{1, 3, 5, 7, 8\}$ and $B = \{2, 5, 7, 10\}$ and you are asked to find the set of elements in A or B the answer is $\{1, 2, 3, 5, 7, 8, 10\}$. (Since you don't know exactly which set of numbers is needed you include all of the numbers from the sets.)

Studyguide for the SAT Skill Review: Sets

Here is some more set notation.

If S a set of elements, then $n(S)$ represents the number of elements in the set.

For example, if $S = \{1, 4, 8, 10\}$, then $n(S) = 4$.

Here is an important relationship to be aware of when calculating the number of elements in a set.

If A and B are sets then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Here is an example of the above relationship.

Let $A = \{1, 3, 6, 9\}$ and $B = \{1, 2, 6, 8, 10\}$ then,

$A \cup B = \{1, 2, 3, 6, 8, 9, 10\}$ and $A \cap B = \{1, 6\}$.

$n(A) = 4$. $n(B) = 5$. $n(A \cup B) = 7$. $n(A \cap B) = 2$.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ when substituting the above numbers we get

$$7 = 4 + 5 - 2$$

$$7 = 7$$



Question 1

If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{4, 5, 6, 7, 8, 9\}$ then what is $A \cap B$?

- (A) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- (B) $\{1, 2, 3, 7, 8, 9\}$
- (C) $\{4\}$
- (D) $\{4, 5, 6\}$
- (E) $\{4, 5\}$

Question 2

If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{4, 5, 6, 7, 8, 9\}$ then what is $A \cup B$?

- (A) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- (B) $\{1, 2, 3, 7, 8, 9\}$
- (C) $\{4\}$
- (D) $\{4, 5, 6\}$
- (E) $\{4, 5\}$

Question 3

If $A = \{1, 3, 5, 7, 9, 15\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{0, 5, 10, 15\}$ then what is $(A \cup B) \cup C$?

- (A) $\{1, 3, 5, 6, 7, 9, 12, 15\}$
- (B) $\{3, 15\}$
- (C) $\{0, 3, 5, 6, 9, 10, 12, 15\}$
- (D) $\{0, 1, 3, 5, 6, 7, 9, 10, 12, 15\}$
- (E) $\{15\}$

Question 4

If $A = \{1, 3, 5, 7, 9, 15\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{0, 5, 10, 15\}$ then what is $(A \cap B) \cap C$?

- (A) $\{1, 3, 5, 6, 7, 9, 12, 15\}$
- (B) $\{3, 15\}$
- (C) $\{0, 3, 5, 6, 9, 10, 12, 15\}$
- (D) $\{0, 1, 3, 5, 6, 7, 9, 10, 12, 15\}$
- (E) $\{15\}$

Studyguide for the SAT Skill Quiz A: Sets

Question 5

If $A = \{1, 3, 5, 7, 9, 15\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{1, 6, 9, 10, 11\}$ then what is $(A \cup B) \cap C$?

- (A) $\{1, 6, 9\}$
- (B) $\{1, 3, 5, 6, 7, 9, 12, 15\}$
- (C) $\{1, 3, 5, 6, 7, 9, 10, 11, 12, 15\}$
- (D) $\{1, 3, 5, 6, 7, 9, 15\}$
- (E) $\{1, 3, 9, 15\}$

Question 6

If $A = \{1, 3, 5, 7, 9, 15\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{1, 6, 9, 10, 11\}$ then what is $A \cup (B \cap C)$?

- (A) $\{1, 6, 9\}$
- (B) $\{1, 3, 5, 6, 7, 9, 12, 15\}$
- (C) $\{1, 3, 5, 6, 7, 9, 10, 11, 12, 15\}$
- (D) $\{1, 3, 5, 6, 7, 9, 15\}$
- (E) $\{1, 3, 9, 15\}$

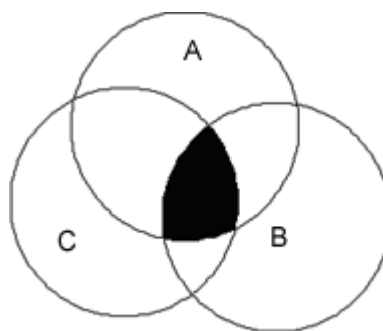
Question 7

If $A = \{1, 3, 5, 7, 9, 15\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{1, 6, 9, 10, 11\}$ then what is $A \cap (B \cup C)$?

- (A) $\{1, 6, 9\}$
- (B) $\{1, 3, 5, 6, 7, 9, 12, 15\}$
- (C) $\{1, 3, 6, 9, 10, 11, 12, 15\}$
- (D) $\{1, 3, 5, 6, 7, 9, 15\}$
- (E) $\{1, 3, 9, 15\}$

Question 8

Which answer choice indicates the shown Venn diagram?

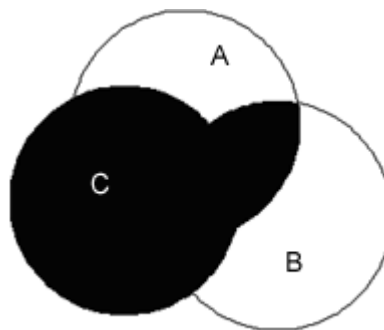


- (A) $(A \cap B) \cup C$
- (B) $(A \cap B) \cap C$
- (C) $A \cup (B \cup C)$
- (D) $A \cup (B \cap C)$
- (E) $(A \cup B) \cap C$

Question 9

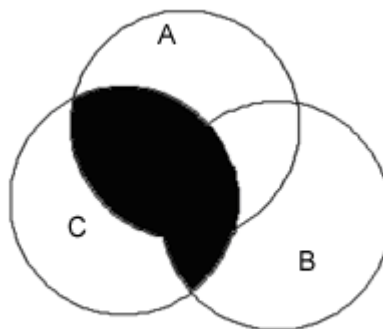
Which answer choice indicates the shown Venn diagram?

- (A) $(A \cap B) \cup C$
- (B) $(A \cap B) \cap C$
- (C) $A \cup (B \cup C)$
- (D) $A \cup (B \cap C)$
- (E) $(A \cup B) \cap C$

**Question 10**

Which answer choice indicates the shown Venn diagram?

- (A) $(A \cap B) \cup C$
- (B) $(A \cap B) \cap C$
- (C) $A \cup (B \cup C)$
- (D) $A \cup (B \cap C)$
- (E) $(A \cup B) \cap C$



Studyguide for the SAT Skill Quiz A: Sets



Hint for Question 1:

The symbol \cap means intersection. The question asks you to find the intersection of sets A and B. The intersection of two sets is the set of elements common to both sets.

Hint for Question 2:

The symbol \cup means union. The question asks you to find the union of sets A and B. The union of two sets is the set of all elements.

Hint for Question 3:

The symbol \cup means union. The question asks you to find the union of sets A, B, and C. The union of two sets is the set of all elements. Find the union of two of the sets and then take that answer with the union of a third set.

Hint for Question 4:

The symbol \cap means intersection. The question asks you to find the intersection of sets A, B and C. The intersection of two sets is the set of elements common to both sets.

Hint for Question 5:

By now you should know that \cup indicates the union of two sets and indicates the intersection of two sets. Work inside the parentheses to find that set first. Take this set and find its intersection with C.

Hint for Question 6:

Work inside the parentheses first. Find the set for $B \cap C$. Take this set and find its union with set A.

Studyguide for the SAT Skill Quiz A: Sets

Hint for Question 7:

Work inside the parentheses first. Find the set for $B \cup C$. Take this set and find its intersection with A.

Hint for Question 8:

This is a time waster. Unless you "see" the answer right away you have to work each answer choice until you arrive at the correct area. Begin with answer choice A and see if the area is correct, if it is not then continue to the next answer choice.

Hint for Question 9:

This is a time waster. Unless you "see" the answer right away you have to work each answer choice until you arrive at the correct area. Begin with answer choice A and see if the area is correct, if it is not then continue to the next answer choice.

Hint for Question 10:

This is a time waster. Unless you "see" the answer right away you have to work each answer choice until you arrive at the correct area. Begin with answer choice A and see if the area is correct, if it is not then continue to the next answer choice.

Studyguide for the SAT Skill Quiz A: Sets



Question 1.

The answer is (D).

Explanation for Question 1:

The intersection of sets A and B is the set of elements common to both sets A and B.

$$A \cap B = \{4, 5, 6\}$$

The answer is (D).

Question 2.

The answer is (A).

Explanation for Question 2:

The union of sets A and B is the set of all elements.

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

The answer is (A).

Question 3.

The answer is (D).

Explanation for Question 3:

The union of sets A and B is the set $\{1, 3, 5, 6, 7, 9, 12, 15\}$. The union of set $\{1, 3, 5, 6, 7, 9, 12, 15\}$ with set C is the set $\{0, 1, 3, 5, 6, 7, 9, 10, 12, 15\}$.

The answer is (D).

Question 4.

The answer is (E).

Explanation for Question 4:

The intersection of sets A and B is the set $\{3, 9, 15\}$. The intersection of set $\{3, 9, 15\}$ with C is the set $\{15\}$.

The answer is (E).

Question 5.

The answer is (A).

Explanation for Question 5:

Work inside the parentheses first. $A \cup B = \{1, 3, 5, 6, 7, 9, 12, 15\}$.
The intersection of $\{1, 3, 5, 6, 7, 9, 12, 15\}$ and C is the set $\{1, 6, 9\}$.
The answer is (A).

Question 6.

The answer is (D).

Explanation for Question 6:

$B \cap C = \{6, 9\}$. The set $\{6, 9\}$ union with set A equals the set $\{1, 3, 5, 6, 7, 9, 15\}$.
The answer is (D).

Question 7.

The answer is (E).

Explanation for Question 7:

$B \cup C = \{1, 3, 6, 9, 10, 11, 12, 15\}$. The set $\{1, 3, 6, 9, 10, 11, 12, 15\}$ intersection with set A equals set $\{1, 3, 9, 15\}$.
The answer is (E).

Question 8.

The answer is (B).

Explanation for Question 8:

Redraw the three intersecting circles. Begin by finding and shading the area indicated inside the parentheses. Take this area and find either its intersection or union with the remaining area. Shade in the solution and see if it fits the diagram. If you are having difficulty solving these types of problems ask your instructor for help.
The answer is (B).

Question 9.

The answer is (A).

Explanation for Question 9:

Redraw the three intersecting circles. Begin by finding and shading the area indicated inside the parentheses. Take this area and find either its intersection or union with the remaining area. Shade in the solution and see if it fits the diagram. If you are having difficulty solving these types of problems ask your instructor for help.

The answer is (A).

Question 10.

The answer is (E).

Explanation for Question 10:

Redraw the three intersecting circles. Begin by finding and shading the area indicated inside the parentheses. Take this area and find either its intersection or union with the remaining area. Shade in the solution and see if it fits the diagram. If you are having difficulty solving these types of problems ask your instructor for help.

The answer is (E).



Question 1

The notation $n(A)$ is used to indicate the number of elements in set A. For example if $A = \{1, 2, 3, 4, 7, 9\}$ then $n(A) = 6$. Use this information to solve the following.

If $R = \{1, 2, 3, 5, 7\}$, $S = \{2, 4, 6, 8, 10\}$ and $T = \{0, 3, 6\}$, what is $n(R)$?

- (A) $\{1, 2, 3, 5, 7\}$
- (B) 4
- (C) 7
- (D) 5
- (E) 0

Question 2

The notation $n(A)$ is used to indicate the number of elements in set A. For example if $A = \{1, 2, 3, 4, 7, 9\}$ then $n(A) = 6$. Use this information to solve the following.

If $R = \{1, 2, 3, 5, 6\}$, $S = \{2, 4, 6, 8, 10\}$ and $T = \{0, 3, 6\}$, what is $n(R \cup S)$?

- (A) 10
- (B) 8
- (C) 6
- (D) 4
- (E) 2

Question 3

The notation $n(A)$ is used to indicate the number of elements in set A. For example if $A = \{1, 2, 3, 4, 7, 9\}$ then $n(A) = 6$. Use this information to solve the following.

If $R = \{1, 2, 3, 5, 7\}$, $S = \{2, 4, 6, 8, 10\}$ and $T = \{0, 3, 7\}$, what is $n(R \cap T)$?

- (A) 10
- (B) 8
- (C) 6
- (D) 4
- (E) 2

Studyguide for the SAT Skill Quiz B: Sets

Question 4

The notation $n(A)$ is used to indicate the number of elements in set A . For example if $A = \{1, 2, 3, 4, 7, 9\}$ then $n(A) = 6$. Use this information to solve the following.

If $R = \{1, 2, 3, 5, 7\}$, $S = \{2, 4, 6, 8, 10\}$ and $T = \{0, 3, 7\}$, what is $n(R) + n(S) + n(T)$?

- (A) 3
- (B) 5
- (C) 7
- (D) 11
- (E) 13

Question 5

If G represents the set of people with green eyes and B represents the set of people with brown hair which of the following sets describes represents $G \cap B$.

- (A) {people with green eyes or brown hair}
- (B) {people with green eyes only}
- (C) {people with brown hair only}
- (D) {people with green eyes and brown hair}
- (E) \emptyset

Question 6

If G represents the set of people with green eyes and B represents the set of people with brown hair which of the following sets describes represents $G \cup B$.

- (A) {people with green eyes or brown hair}
- (B) {people with green eyes only}
- (C) {people with brown hair only}
- (D) {people with green eyes and brown hair}
- (E) \emptyset

Question 7

A survey was conducted among seniors at Central High School for the cafeteria. The survey found that 50 seniors liked pizza for lunch, 90 seniors liked hamburgers for lunch and 20 seniors liked both hamburgers and pizza for lunch. How many seniors only liked pizza for lunch?

- (A) 140
- (B) 120
- (C) 70
- (D) 50
- (E) 30

Question 8

A survey was conducted among seniors at Central High School for the cafeteria. The survey found that 50 seniors liked pizza for lunch, 90 seniors liked hamburgers for lunch and 20 seniors liked both hamburgers and pizza for lunch. How many seniors liked pizza or hamburgers for lunch?

- (A) 140
- (B) 120
- (C) 70
- (D) 50
- (E) 30

Question 9

Yesterday Officer Jones issued tickets to 16 drivers. She issued 12 tickets for speeding and 7 tickets for driving while using a cell phone. How many drivers received tickets for both speeding and driving using a cell phone?

- (A) 3
- (B) 4
- (C) 5
- (D) 9
- (E) 19

Question 10

A survey of 100 students who liked either cola flavored or orange flavored soda was taken at Central High School. 60 of these students liked cola flavored sodas and 55 liked orange flavored sodas. How many students like both cola flavored and orange flavored sodas?

- (A) 5
- (B) 15
- (C) 40
- (D) 45
- (E) 115

Studyguide for the SAT Skill Quiz B: Sets



Hint for Question 1:

Use the explanation for what $n(A)$ represents. $n(R)$ represents the number of elements in R . Count how many elements are in R .

Hint for Question 2:

First find the set $R \cup S$. Count the number of elements in this set to get the answer.

Hint for Question 3:

Find $R \cap T$ and count the number of elements in the set.

Hint for Question 4:

Find the sum of the number of elements in each set.

Hint for Question 5:

What connecting word represents the intersection of two sets? If you've forgotten reread the lesson.

Hint for Question 6:

What connecting word represents the union of two sets? If you've forgotten reread the lesson.

Hint for Question 7:

Seniors that only liked pizza would not also like hamburgers.

Hint for Question 8:

A common mistake is to add the number of seniors who like pizza with the number of seniors who like hamburgers. When you do this you include the number of seniors who like pizza and hamburgers twice since they are included in the numbers of both sets. Think what is needed to not include the senior pizza hamburger set twice.

Hint for Question 9:

Remember the relationship $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Let $n(A)$ be the number of tickets issued for speeding and $n(B)$ be the number of tickets issued for using a cell phone. $n(A \cup B)$ would represent the number of drivers ticketed and $n(A \cap B)$ would represent the number of drivers ticketed for both speeding and driving using a cell phone. Substitute in the correct numbers and solve.

Hint for Question 10:

Remember the relationship $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Let $n(A)$ be the number of students liking cola flavored sodas and $n(B)$ be the number of students liking orange flavored sodas. $n(A \cup B)$ would represent the number of students liking either cola or orange flavored sodas and $n(A \cap B)$ would represent the number of students liking both cola and orange flavored sodas. Substitute in the correct numbers and solve.

Studyguide for the SAT Skill Quiz B: Sets



Question 1.

The answer is (D).

Explanation for Question 1:

The number of elements in R is 5 so $n(r) = 5$.

The answer is (D).

Question 2.

The answer is (B).

Explanation for Question 2:

$R \cup S = \{1, 2, 3, 4, 5, 6, 8, 10\}$. The number of elements in this set is 8 so $n(R \cup S) = 8$.

The answer is (B).

Question 3.

The answer is (E).

Explanation for Question 3:

$R \cap T = \{3, 7\}$. The number of elements in $R \cap T$ is 2 so $n(R \cap T) = 2$.

The answer is (E).

Question 4.

The answer is (E).

Explanation for Question 4:

$n(R) = 5$. $n(S) = 5$. $n(T) = 3$. $n(R) + n(S) + n(T) = 5 + 5 + 3 = 13$

The answer is (E).

Question 5.

The answer is (D).

Explanation for Question 5:

The connecting word that represents intersection is the word and. $G \cap B$ is the set of people having green eyes and brown hair.

The answer is (D).

Question 6.

The answer is (A).

Explanation for Question 6:

The connecting word that represents union is the word or. $G \cup B$ is the set of people having green eyes or brown hair.

The answer is (A).

Question 7.

The answer is (E).

Explanation for Question 7:

Subtract the number of seniors that liked both pizza and hamburgers from the number of seniors that liked pizza. $50 - 20 = 30$. A common mistake is to assume that the 50 seniors that liked pizza only liked pizza. Among the 50 students that liked pizza could also be some that liked hamburgers.

The answer is (E).

Question 8.

The answer is (B).

Explanation for Question 8:

The answer can be found by adding the number of seniors that like pizza with the number of seniors that like hamburgers and then subtracting from the sum the number of seniors that like both pizza and hamburgers.

$$50 + 90 - 20 = 120.$$

There is a formula that will prove beneficial in solving this type of problem.

Let $n(P)$ be the number of seniors that like pizza and $n(H)$ be the number of seniors that like hamburgers then

$$n(P \cup H) = n(P) + n(H) - n(P \cap H)$$

Remember the set $P \cup H$ represents the students who like pizza or hamburgers and the set $P \cap H$ represents students who like pizza and hamburgers.

The answer is (B).

Question 9.

The answer is (A).

Explanation for Question 9:

Look at the above hint and use the following relationship.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

$$n(A \cup B) = 16. \quad n(A) = 12. \quad n(B) = 7. \quad \text{You are solving for } n(A \cap B).$$

$$16 = 12 + 7 - n(A \cap B).$$

$$16 = 19 - n(A \cap B).$$

$$-3 = -n(A \cap B).$$

$$n(A \cap B) = 3$$

The answer is (A).

Question 10.

The answer is (B).

Explanation for Question 10:

Look at the above hint and use the following relationship.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

$n(A \cup B) = 100$. $n(A) = 60$. $n(B) = 55$. You are solving for $n(A \cap B)$.

$$100 = 60 + 55 - n(A \cap B).$$

$$100 = 115 - n(A \cap B).$$

$$-15 = -n(A \cap B).$$

$$n(A \cap B) = 15$$

The answer is (B).



Question 1

If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{\text{prime numbers}\}$ then what is $A \cap B$?

- (A) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- (B) $\{1, 3, 5, 7, 9\}$
- (C) $\{1, 2, 3, 5, 7\}$
- (D) $\{2, 3, 5, 7, 9\}$
- (E) $\{2, 3, 5, 7\}$

Question 2

If $A = \{1, 3, 5\}$ and $B = \{3, 4, 6, 8\}$ then what is $A \cup B$?

- (A) $\{1, 4, 5, 6, 8\}$
- (B) $\{1, 3, 4, 5, 6, 8\}$
- (C) $\{3\}$
- (D) $\{4, 6, 8\}$
- (E) \emptyset

Question 3

If $A = \{1, 3, 5, 7, 9, 15\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{0, 5, 10, 15\}$ then what is $(A \cap B) \cup C$?

- (A) $\{1, 3, 5, 6, 7, 9, 12, 15\}$
- (B) \emptyset
- (C) $\{0, 3, 5, 9, 10, 15\}$
- (D) $\{0, 1, 3, 5, 6, 7, 9, 10, 12, 15\}$
- (E) $\{15\}$

Question 4

If $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$ then what is $A \cap B$?

- (A) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- (B) $\{1, 3, 5, 7, 9\}$
- (C) $\{2, 4, 6, 8\}$
- (D) $\{1, 2, 3, 4, 5\}$
- (E) \emptyset

Studyguide for the SAT Skill Quiz C: Sets

Question 5

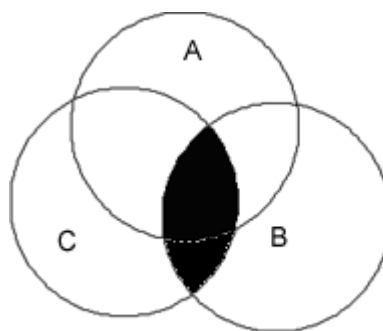
If G represents the set of people having green eyes and R represents the set of people having red hair then which of the following best describes $G \cap R$?

- (A) {people with green eyes only}
- (B) {people with red hair only}
- (C) {people with green eyes and red hair}
- (D) {people with red eyes and green hair}
- (E) {people with green eyes or red hair}

Question 6

Which answer choice indicates the shown Venn diagram?

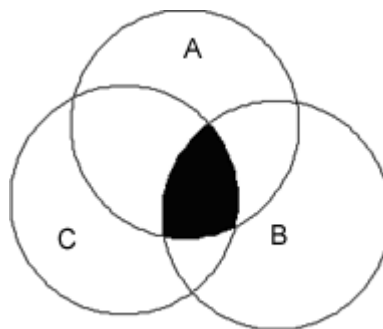
- (A) $A \cap B$
- (B) $(A \cap B) \cap C$
- (C) $B \cup C$
- (D) $B \cap C$
- (E) $(A \cup B) \cap C$



Question 7

Which answer choice indicates the shown Venn diagram?

- (A) $(A \cap B) \cup C$
- (B) $(A \cup B) \cap C$
- (C) $A \cup (B \cap C)$
- (D) $A \cup (B \cap C)$
- (E) $(A \cap B) \cap C$



Studyguide for the SAT Skill Quiz C: Sets

Question 8

The notation $n(A)$ is used to indicate the number of elements in set A . For example if $A = \{1, 2, 3, 4, 7, 9\}$ then $n(A) = 6$. Use this information to solve the following.

If $R = \{1, 2, 3, 5, 7\}$, $S = \{2, 4, 6, 8, 10\}$ and $T = \{0, 3, 6\}$, what is $n(R \cup (S \cup T))$?

- (A) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 10\}$
- (B) 3
- (C) 5
- (D) 9
- (E) 10

Question 9

A survey was conducted among seniors at Central High School for the cafeteria. The survey found that 80 seniors liked pizza for lunch, 55 seniors liked hamburgers for lunch and 40 seniors liked both hamburgers and pizza for lunch. How many seniors liked pizza or hamburgers for lunch?

- (A) 175
- (B) 135
- (C) 95
- (D) 40
- (E) 15

Question 10

For a week Officer Jones issued tickets to 43 drivers. She issued 35 tickets for speeding and 15 tickets for driving while using a cell phone. How many drivers received tickets for both speeding and driving using a cell phone?

- (A) 93
- (B) 78
- (C) 50
- (D) 7
- (E) 0

Studyguide for the SAT Skill Quiz C: Sets



Hint for Question 1:

The symbol \cap means intersection. The question asks you to find the intersection of sets A and B. The intersection of two sets is the set of elements common to both sets. Be careful the smallest prime number is 2.

Hint for Question 2:

The symbol \cup means union. The question asks you to find the union of sets A and B. The union of two sets is the set of all elements.

Hint for Question 3:

Work inside the parentheses first. The symbol \cap means intersection. Find the intersection of sets A and B. Take this set with the union of set C.

Hint for Question 4:

The symbol \cap means intersection. The question asks you to find the intersection of sets A and B. The intersection of two sets is the set of elements common to both sets.

Hint for Question 5:

Remember the word used to indicate the intersection of sets is the word and.

Hint for Question 6:

Remember this is a time waster. On the SAT you may wish to skip this one and return to it later. One approach is to find the solution for each of the answer choices until you get a figure that matches the above figure.

Hint for Question 7:

You've seen this one before.

This is a time waster. Unless you "see" the answer right away you have to work each answer choice until you arrive at the correct area.

Begin with answer choice A and see if the area is correct, if it is not then continue to the next answer choice.

Hint for Question 8:

Use the explanation for what $n(A)$ represents. Find the set $R \cup (S \cup T)$ and count the number of elements in the set.

Hint for Question 9:

A common mistake is to add the number of seniors who like pizza with the number of seniors who like hamburgers. When you do this you include the number of seniors who like pizza and hamburgers twice since they are included in the numbers of both sets. Think what is needed to not include the senior pizza hamburger set twice.

Hint for Question 10:

Remember the relationship $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Let $n(A)$ be the number of tickets issued for speeding and $n(B)$ be the number of tickets issued for using a cell phone. $n(A \cup B)$ would represent the number of drivers ticketed and $n(A \cap B)$ would represent the number of drivers ticketed for both speeding and driving using a cell phone.

Substitute in the correct numbers and solve.



Question 1.

The answer is (E).

Explanation for Question 1:

The intersection of sets A and B is the set of elements common to both sets A and B.

Set B = {2, 3, 5, 7, 11, 13, ...}. 1 is not a prime number.

The answer is (E).

Question 2.

The answer is (B).

Explanation for Question 2:

The union of sets A and B is the set of all elements.

A \cup B = {1, 3, 4, 5, 6, 8}.

The answer is (B).

Question 3.

The answer is (C).

Explanation for Question 3:

The intersection of sets A and B is the set {3, 9, 15}. The union of set {3, 9, 15} with set C is the set {0, 3, 5, 9, 10, 15}.

The answer is (C).

Question 4.

The answer is (E).

Explanation for Question 4:

Sets A and B have no elements in common. Their intersection is the empty or null set.

The answer is (E).

Question 5.

The answer is (C).

Explanation for Question 5:

Remember the word used to indicate the intersection of sets is the word and. $G \cap R$ is the people with green eyes and red hair.

The answer is (C).

Question 6.

The answer is (D).

Explanation for Question 6:

Redraw the three intersecting circles. Begin by finding and shading the area indicated inside the parentheses. Take this area and find either its intersection or union with the remaining area. Shade in the solution and see if it fits the diagram. If you are having difficulty solving these types of problems ask your instructor for help.

The answer is (D).

Question 7.

The answer is (E).

Explanation for Question 7:

Redraw the three intersecting circles. Begin by finding and shading the area indicated inside the parentheses. Take this area and find either its intersection or union with the remaining area. Shade in the solution and see if it fits the diagram. If you are having difficulty solving these types of problems ask your instructor for help.

The answer is (E).

Question 8.

The answer is (E).

Explanation for Question 8:

$$R \cup (S \cup T) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 10\}. \quad n(R \cup (S \cup T)) = 10$$

The answer is (E).

Question 9.

The answer is (C).

Explanation for Question 9:

The answer can be found by adding the number of seniors that like pizza with the number of seniors that like hamburgers and then subtracting from the sum the number of seniors that like both pizza and hamburgers.

$$80 + 55 - 40 = 95.$$

There is a formula that will prove beneficial in solving this type of problem.

Let $n(P)$ be the number of seniors that like pizza and $n(H)$ be the number of seniors that like hamburgers then

$$n(P \cup H) = n(P) + n(H) - n(P \cap H)$$

Remember the set $P \cap H$ represents the students who like pizza and hamburgers and the set $P \cup H$ represents students who like pizza or hamburgers.

The answer is (C).

Question 10.

The answer is (D).

Explanation for Question 10:

Look at the above hint and use the following relationship.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

$$n(A \cup B) = 43. \quad n(A) = 35. \quad n(B) = 15. \quad \text{You are solving for } n(A \cap B).$$

$$43 = 35 + 15 - n(A \cap B).$$

$$43 = 50 - n(A \cap B).$$

$$-7 = -n(A \cap B).$$

$$n(A \cap B) = 7$$

The answer is (D).

Here is another way to think about the solution. Officer Jones gave 50 tickets $(35 + 15)$ to 43 drivers. 7 drivers $(50 - 43)$ must have received 2 tickets.

Math Lesson #6

Functions

- *Function Substitution*
- *Substitution*
- *Functions*

Triumph College Admissions

 **Skill Lesson**

 **Quiz Time!**

 **Hints!**

 **Answers!**

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FUNCTION SUBSTITUTION

Each SAT has math questions that involve a special symbol defined by some function stated in the question. You are expected to apply the given definition of the function to solve the problem.

Although the symbols vary from test to test, and the symbols often look strange, the questions are generally fairly easy if you simply apply the definition stated in the question.

Example 1:

If $a \# b = 5a - 2b$, what is the value of $3 \# 4$?

Solution:

Substitute 3 for a and 4 for b . Always be sure to watch the order of the variables and substitute the numbers in the exact place of the variables.
 $3 \# 4 = 5(3) - 2(4) = 15 - 8 = 7$

Example 2:

If $x @ y = y^2 - x$, what is the value of $3 @ 4$?

Solution:

Substitute 3 for x and 4 for y . $3 @ 4 = 4^2 - 3 = 16 - 3 = 13$



Question 1 refers to the following definition.

$$\boxed{n} = n^2 + 2n - 3$$

For example,

$$\boxed{4} = 4^2 + 2(4) - 3 = 16 + 8 - 3 = 21$$

Question 1

$$\boxed{5} =$$

(A) 35

(D) 17

(B) 32

(E) 12

(C) 23

Question 2 refers to the following definition.

$$\boxed{n} = n^2 + 2n - 3$$

For example,

$$\boxed{4} = 4^2 + 2(4) - 3 = 16 + 8 - 3 = 21$$

Question 2

$$\boxed{x^2} =$$

(A) $x^2 + 2x - 3$


(D) $3x^3 - 3$

(B) $x^4 + 2x - 3$

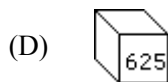
(E) $3x^6 - 3$

(C) $x^4 + 2x^2 - 3$

Question 3

If  represents x^2 , then the area of a square with

sides of length 25 is represented by



Question 4 refers to the following definition.

n is an integer greater than 1.

F_n denotes the smallest positive integer factor of n not equal to 1.

Question 4

$F_{24} =$

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Studyguide for the SAT Skill Quiz A: Function Substitution

Question 5 refers to the following definition.

n is an integer greater than 1.

F_n denotes the smallest positive integer factor of n not equal to 1.

Question 5

$$F_{51} =$$

- (A) 2
- (B) 3
- (C) 11
- (D) 17
- (E) 51

Question 6

The symbol $\boxed{x} = x^2 - 2x$

What is the value of $\boxed{5}$?

- (A) 25
- (B) 15
- (C) 10
- (D) 5
- (E) -5

Question 7

The symbol $\boxed{x} = x^2 - 2x$

What is the value of $\boxed{6} - \boxed{3}$?

- (A) 3
- (B) 9
- (C) 21
- (D) 27
- (E) 33

Question 8

If $x \% y = 2x + y$, what is the value of $3 \% 2$?

- (A) 5
- (B) 6
- (C) 7
- (D) 8
- (E) 10

Question 9

If $a @ b = ab + a$, which of the following is equal to 20?

- (A) $4 @ 2$
- (B) $5 @ 3$
- (C) $10 @ 2$
- (D) $3 @ 5$
- (E) $2 @ 10$

Question 10

If $\boxed{a} = \sqrt{a} - 1$, what is the value of $\boxed{100}$?

- (A) 9
- (B) 10
- (C) 99
- (D) 100
- (E) 101

**Hint for Question 1:**

Substitute 5 for n in the definition.

Hint for Question 2:

Substitute x^2 for n in the definition.

Hint for Question 3:

Don't be manipulated by the question.
What is the area of a square?

 $= x^2$

Hint for Question 4:

Other than 1, what is the smallest positive factor of 24?

Hint for Question 5:

Other than 1, what is the smallest factor of 51?

Hint for Question 6:

Substitute 5 for x in the expression $x^2 - 2x$.

Hint for Question 7:

Substitute 6 for x and then 3 for x and subtract your answers.

Hint for Question 8:

Substitute 3 for x , 2 for y , and evaluate the expression.

Hint for Question 9:

You have to evaluate each answer choice. Substitute the first number for a and the second number for b .

Hint for Question 10:

Substitute 100 for a in the expression and evaluate.



Question 1.

The answer is (B).

Explanation for Question 1:

$$\boxed{5} = 5^2 + 2(5) - 3 = 25 + 10 - 3 = 32$$

The answer is (B).

Question 2.

The answer is (C).

Explanation for Question 2:

$$\boxed{\frac{2}{x}} = x^4 + 2x^2 - 3$$

The answer is (C).

Question 3.

The answer is (B).

Explanation for Question 3:

$$A = s^2 = 25^2 = \boxed{25}$$

The answer is (B).

Question 4.

The answer is (A).

Explanation for Question 4:

The smallest integer factor of 24 (not equal to 1) is 2.

The answer is (A).

Question 5.

The answer is (B).

Explanation for Question 5:

The factors of 51 are 1, 3, 17, 51, so

$$\frac{F}{51} = 3$$

The answer is (B).

Question 6.

The answer is (B).

Explanation for Question 6:

Substitute 5 for x. $5^2 - 2(5) = 25 - 10 = 15$. The answer is (B).

Question 7.

The answer is (C).

Explanation for Question 7:

$$6^2 - 2(6) = 36 - 12 = 24, \quad 3^2 - 2(3) = 9 - 6 = 3.$$
$$24 - 3 = 21. \text{ The answer is (C).}$$

Question 8.

The answer is (D).

Explanation for Question 8:

$$3 \% 2 = 2(3) + 2 = 6 + 2 = 8. \text{ The answer is (D).}$$

Question 9.

The answer is (B).

Explanation for Question 9:

$$4 @ 2 = 4(2) + 4 = 12$$
$$5 @ 3 = 5(3) + 5 = 20. \text{ Done. The answer is (B).}$$

Question 10.

The answer is (A).

Explanation for Question 10:

$$\sqrt{100} - 1 = 10 - 1 = 9. \text{ The answer is (A).}$$



Question 1

Given: $\begin{array}{c} \text{a} \\ \diagup \quad \diagdown \\ \text{d} \quad \text{b} \\ \diagdown \quad \diagup \\ \text{c} \end{array} = ac - bd$

What is the value of $\begin{array}{c} 2 \\ \diagup \quad \diagdown \\ 5 \quad 3 \\ \diagdown \quad \diagup \\ 4 \end{array} ?$

- (A) 23
- (B) 7
- (C) 0
- (D) -7
- (E) -23

Question 2

Given: $\begin{array}{c} \text{a} \\ \diagup \quad \diagdown \\ \text{d} \quad \text{b} \\ \diagdown \quad \diagup \\ \text{c} \end{array} = ac - bd$

What is the value of $\begin{array}{c} 3 \\ \diagup \quad \diagdown \\ 4 \quad 2 \\ \diagdown \quad \diagup \\ 1 \end{array} + \begin{array}{c} 6 \\ \diagup \quad \diagdown \\ 3 \quad 2 \\ \diagdown \quad \diagup \\ 1 \end{array}$

- (A) -5
- (B) 5
- (C) 17
- (D) 23
- (E) 33

Question 3

Given: $a \$ b = 3a - 2b$

What is the value of $5 \$ 3$?

- (A) 2
- (B) 8
- (C) 9
- (D) 19
- (E) 24

Question 4

Given: $a \$ b = 3a - 2b$

What is the value of $(3 \$ 2) + (4 \$ 1)$?

- (A) $5 \$ 1$
- (B) $7 \$ 3$
- (C) $6 \$ 2$
- (D) $2 \$ 6$
- (E) $3 \$ 7$

Question 5

Given: $a \# b = a + b + 2$

Evaluate $5 \# 1$.

- (A) 5
- (B) 6
- (C) 7
- (D) 8
- (E) 9

Question 6

If $a \# b = a + b + 2$, which of the following must be true?

- I. $a \# b = b \# a$
- II. $2(a \# b) = 2a \# 2b$
- III. $a \# (b \# c) = (a \# b) \# c$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) I and III

Question 7

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be defined as $ad + cb$. If $x = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$,

then what is the value of $\begin{bmatrix} 1 & 3 \\ 5 & x \end{bmatrix}$?

- (A) 9
- (B) 14
- (C) 17
- (D) 24
- (E) 29

Question 8

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be defined as $ad + cb$ and $\begin{bmatrix} 2 & 5 \\ 3 & x \end{bmatrix} = 27$.

What is the value of x ?

- (A) 6
- (B) 7.5
- (C) 8
- (D) 12
- (E) 22

Question 9

Let $a \# b = 2a + 3b$. What is the value of $x \# x$?

- (A) $5x^2$
- (B) $6x^2$
- (C) $5x$
- (D) $6x$
- (E) $10x$

Question 10

Let $a \# b = 3a + b$. What is the value of $x \# -x$?

- (A) $-2x$
- (B) $2x$
- (C) 0
- (D) $-3x^2$
- (E) $3x^2$

Studyguide for the SAT Skill Quiz B: Function Substitution



Hint for Question 1:

Substitute into the expression the numbers for a, b, c, and d.

Hint for Question 2:

Evaluate each circle by substituting into the expression the numbers for a, b, c, and d.

Hint for Question 3:

Substitute 5 for a and 3 for b.

Hint for Question 4:

Evaluate each expression and then add.

Hint for Question 5:

Substitute 5 for a and 1 for b.

Hint for Question 6:

Instead of using a's and b's try some numbers. For example let $a = 4$ and $b = 5$, then see which of the equations are true. Pick some numbers that are not in the equations.

Hint for Question 7:

First solve for x and then substitute that value for x in the last box.

Hint for Question 8:

Write the expression for the box and set it equal to 27. Solve for x.

Hint for Question 9:

Substitute x for both a and b.

Hint for Question 10:

Substitute the x for a and -x for b.



Question 1.

The answer is (D).

Explanation for Question 1:

$a = 2$, $b = 3$, $c = 4$, and $d = 5$. Substitute into the expression these numbers for a , b , c , and d . $2(4) - 3(5) = 8 - 15 = -7$ The answer is (D).

Question 2.

The answer is (A).

Explanation for Question 2:

$3(1) - 2(4) = 3 - 8 = -5$, and $6(1) - 2(3) = 6 - 6 = 0$, then $-5 + 0 = -5$. The answer is (A).

Question 3.

The answer is (C).

Explanation for Question 3:

$3(5) - 2(3) = 15 - 6 = 9$. The answer is (C).

Question 4.

The answer is (B).

Explanation for Question 4:

$(3(3) - 2(2)) + (3(4) - 2(1)) = (9 - 4) + (12 - 2) = 5 + 10 = 15$. Now find which answer choice is equal to 15.

(A) $3(5) - 2(1) = 15 - 2 = 13$

(B) $3(7) - 2(3) = 21 - 6 = 15$

The answer is (B).

Question 5.

The answer is (D).

Explanation for Question 5:

$5 + 1 + 2 = 8$. The answer is (D).

Question 6.

The answer is (E).

Explanation for Question 6:

Let $a = 4$, $b = 5$, and $c = 6$. Then

I. $4 \# 5 = 5 \# 4$, so $4 + 5 + 2 = 5 + 4 + 2$, so $11 = 11$. True.

II. $2(4 \# 5) = 2(4) \# 2(5)$, so $2(4 + 5 + 2) = 8 \# 10$

$2(11) = 8 + 10 + 2$

$22 = 20$. False.

III. $4 \# (5 \# 6) = (4 \# 5) \# 6$, so $4 \# (5 + 6 + 2) = (4 + 5 + 2) \# 6$

$4 \# 13 = 11 \# 6$

$4 + 13 + 2 = 11 + 6 + 2$

$19 = 19$. True.

I and III are true. The answer is (E).

Question 7.

The answer is (E).

Explanation for Question 7:

First, solve for x . $x = 1(2) + 4(3)$, so $x = 2 + 12$, so $x = 14$. Substitute 14 in for x in the last box and evaluate. $1(14) + 5(3) = 14 + 15 = 29$. The answer is (E).

Question 8.

The answer is (A).

Explanation for Question 8:

$$2(x) + 3(5) = 27$$

$$2x + 15 = 27$$

$$2x = 12$$

$x = 6$. The answer is (A).

Question 9.

The answer is (C).

Explanation for Question 9:

$$2x + 3x = 5x. \text{ The answer is (C).}$$

Question 10.

The answer is (B).

Explanation for Question 10:

$$x \# -x = 3(x) + (-x) = 3x - x = 2x$$

The answer is (B).



Question 1

Given: $*a = a^2 + 1$

Which of the following is equal to 82?

- (A) $*81$
- (B) $*10$
- (C) $*9$
- (D) $*8$
- (E) $*6$

Question 2

Given: $*a = a^2 + 1$

$*10 - *8$ is equal to which of the following?

- (A) $*6$
- (B) $*6 - 1$
- (C) $*2$
- (D) $*6 + 1$
- (E) $*2 + 1$

Question 3

Let $@x$ be defined for any positive integer as the number obtained by writing the digits of x in reverse order.

Example: $@3 = 3$ and $@179 = 971$

$@13 - @14$ is equal to which of the following?

- (A) 1
- (B) 10
- (C) 27
- (D) -10
- (E) -1

Studyguide for the SAT Skill Quiz C: Function Substitution

Question 4

Let $@x$ be defined for any positive integer as the number obtained by writing the digits of x in reverse order.

Example: $@3 = 3$ and $@179 = 971$

$@123 - @421$ is equal to which of the following?

- (A) $@197$
- (B) $@971$
- (C) $@791$
- (D) $@298$
- (E) $@892$

Question 5

If $x \# y = 2xy$, what is the value of $4 \# 3$?

- (A) 7
- (B) 11
- (C) 12
- (D) 24
- (E) 26

Question 6

Given: $a \# b = 2ab$

Evaluate $5 \# 3$.

- (A) 5
- (B) 15
- (C) 24
- (D) 30
- (E) 60

Question 7

If $a \# b = 2ab$, which of the following must be true.

- I. $a \# b = b \# a$
- II. $2(a \# b) = 2a \# 2b$
- III. $a \# (b \# c) = (a \# b) \# c$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) I and III

Question 8

Let $a \# b = 2ab$. What is the value of $3x \# x$?

- (A) $5x^2$
- (B) $6x^2$
- (C) $5x$
- (D) $6x$
- (E) $10x$

Question 9

Let $a \# b = 3ab$. What is the value of $xy \# x$?

- (A) $3x^2y$
- (B) $6xy$
- (C) $3abx^2y$
- (D) $4x + y$
- (E) $10x$

Question 10

Let $a \# b = a^b$. What is the value of $3x \# 3$?

(A) $6x$

(B) $9x$

(C) $9x^3$

(D) $27x$

(E) $27x^3$

Studyguide for the SAT Skill Quiz C: Function Substitution



Hint for Question 1:

Substitute the number in each answer choice for a in the expression and simplify.

Hint for Question 2:

Evaluate $*10 - *8$ and then see which of the answer choices is equal to it.

Hint for Question 3:

Be sure you understand the directions. Reverse the order of the digits.

Hint for Question 4:

Find the answer to $@123 - @421$. The answer choices have to have their digits reversed to equal this answer.

Hint for Question 5:

Substitute 4 for x and 3 for y .

Hint for Question 6:

Substitute 5 for a and 3 for b .

Hint for Question 7:

Pick some numbers to use for a , b , and c . Then see which of the equations are true.

Hint for Question 8:

Substitute $3x$ for a and y for b .

Hint for Question 9:

Substitute $3x$ for a and y for b .

Hint for Question 10:

Substitute $3x$ for a and 3 for b .



Question 1.

The answer is (C).

Explanation for Question 1:

Evaluate each answer choice.

(A) $81^2 + 1 =$ (too big)

(B) $10^2 + 1 = 100 + 1 = 101$. No.

(C) $9^2 + 1 = 81 + 1 = 82$. Yes. The answer is (C).

Question 2.

The answer is (B).

Explanation for Question 2:

$*10 - *8 = 10^2 + 1 - (8^2 + 1) = 100 + 1 - (64 + 1) = 101 - 65 = 36$. Now which of the answer choices is equal to 36?

(A) $6^2 + 1 = 36 + 1 = 37$. No.

(B) (You've already evaluated *6 just subtract 1 from it.) $37 - 1 = 36$.

The answer is (B).

Question 3.

The answer is (D).

Explanation for Question 3:

$@13 - @14 = 31 - 41 = -10$. The answer is (D).

Question 4.

The answer is (C).

Explanation for Question 4:

$@123 - @421 = 321 - 124 = 197 = @791$. The answer is (C).

(Just reverse the order of 197.)

Question 5.

The answer is (D).

Explanation for Question 5:

$4 \# 3 = 2(4)(3) = 24$. The answer is (D).

Question 6.

The answer is (D).

Explanation for Question 6:

This is the same problem as number 5. It just uses different variables.

$5 \# 3 = 2(5)(3) = 30$. The answer is (D).

Studyguide for the SAT Skill Quiz C: Function Substitution

Question 7.

The answer is (E).

Explanation for Question 7:

Let $a = 3$, $b = 4$, and $c = 5$.

I. $3 \# 4 = 4 \# 3$, so $2(3)(4) = 2(4)(3)$, so $24 = 24$. True

II. $2(3 \# 4) = 2(3) \# 2(4)$, so $2(2)(3)(4) = 6 \# 8$, so $48 = 2(6)(8)$
 $48 = 96$. False

III. $3 \# (4 \# 5) = (3 \# 4) \# 5$, so $3 \# (2(4)(5)) = (2(3)(4)) \# 5$
 $3 \# 40 = 24 \# 5$

$$2(3)(40) = 2(24)(5)$$

$$240 = 240. \text{ True.}$$

Statements I and III are true. The answer is (E).

Question 8.

The answer is (B).

Explanation for Question 8:

$$3x \# x = 2(3x)(x) = 6x^2 \text{ The answer is (B).}$$

Question 9.

The answer is (A).

Explanation for Question 9:

$$a \# b = 3ab$$

$$xy \# x = 3(xy)(x) = 3x^2y$$

The answer is (A).

Question 10.

The answer is (E).

Explanation for Question 10:

$$a \# b = a^b$$

$$3x \# 3 = 3x^3 = 3x \times 3x \times 3x = 27x^3$$

The answer is (E).



SUBSTITUTION

There may be a few problems on the SAT that require you to substitute a number for a variable in an equation or expression. Substitution problems should not be that difficult and can be made easier if you remember to include parentheses around the number you are substituting.

For example, if $f(x) = x^3 + 2x^2 - 4x + 7$, find $f(3)$.

Solution:

You need to substitute the number 3 for x in the equation

$$f(x) = x^3 + 2x^2 - 4x + 7.$$

Place parentheses around every 3 you substitute for x . Your equation should look like the following.

$f(3) = (3)^3 + 2(3)^2 - 4(3) + 7$. If you can do order of operations the solution should be easy.

$$\begin{aligned} f(3) &= 27 + 2(9) - 4(3) + 7 \\ &= 27 + 18 - 12 + 7 \\ &= 40. \end{aligned}$$

If you have difficulty with order of operations you may use a calculator that allows you to substitute for x after entering the equation. The problem would take longer to solve using the calculator.

To solve substitution problems replace the variable with the number and simplify.



Question 1

What is the value of $x^2 - 2x + 3$, when $x = 3$?

- (A) 3
- (B) 4
- (C) 6
- (D) 15
- (E) 24

Question 2

Given $f(x) = (x + 2)(x - 4) + 2x$, find $f(-4)$.

- (A) -24
- (B) -8
- (C) 0
- (D) 8
- (E) 16

Question 3

If $G(x) = 100x - 5$, find $G(10)$.

- (A) 95
- (B) 105
- (C) 500
- (D) 995
- (E) 1,005

Question 4

Given $r(x) = 2x + 1$ and $s(x) = 3x - 4$, find $r(3) + s(2)$.

- (A) 9
- (B) 10
- (C) 14
- (D) 17
- (E) 18

Question 5

If $p(x) = 2x + 1$ and $r(x) = x^2$, what is $p(r(3))$?

- (A) 13
- (B) 16
- (C) 19
- (D) 49
- (E) 53

Question 6

If $g(x) = x^2 + 2x + 3$, find $g(-3)$.

- (A) -12
- (B) 0
- (C) 3
- (D) 6
- (E) 18

Question 7

Evaluate $4a^2b^3$, if $a = 3$ and $b = 2$.

- (A) 24
- (B) 144
- (C) 216
- (D) 288
- (E) 1,152

Question 8

What is the value of $(x + y)(x^2 - xy + 4)$, when $x = 2$ and $y = 3$?

- (A) -5
- (B) -2
- (C) 1
- (D) 2
- (E) 10

Studyguide for the SAT Skill Quiz A: Substitution

Question 9

If $y = |x| - 2x - x^2$, what is the value of y when $x = -2$?

- (A) -6
- (B) -2
- (C) 1
- (D) 2
- (E) 6

Question 10

If $F(x) = \frac{x+3}{x-2}$, find $F(1)$.

- (A) -7
- (B) -4
- (C) 0
- (D) 1
- (E) 7

Studyguide for the SAT Skill Quiz A: Substitution



Hint for Question 1:

Substitute a 3 for the variable x in the expression $x^2 - 2x + 3$.
If you use parentheses around the 3 you
may eliminate some easy mistakes.

Hint for Question 2:

To find $f(-4)$ substitute a (-4) in for x in the function
 $f(x) = (x + 2)(x - 4) + 2x$.
Remember to simplify inside the parentheses first.

Hint for Question 3:

Substitute 10 for x in $G(x) = 100x - 5$.

Hint for Question 4:

Substitute a 3 for the x in $r(x)$ and a 2 for x in $s(x)$.
Evaluate each function and add.

Hint for Question 5:

Be careful. First evaluate $r(3)$.
Take the answer for $r(3)$ and substitute it for the x in $p(x)$.
This is a composition of functions problem,
not a multiplication problem.

Hint for Question 6:

Substitute -3 for x in the function $g(x)$.

Studyguide for the SAT Skill Quiz A: Substitution

Hint for Question 7:

Substitute 3 for a and 2 for b.
Simplify exponents before multiplying.

Hint for Question 8:

Substitute a 2 for x and a 3 for y in the expression.

Hint for Question 9:

Substitute a (-2) for x in the equation.
Be careful of the absolute value.

Hint for Question 10:

Substitute a 1 for x in function F.



Question 1.

The answer is (C).

Explanation for Question 1:

Substitute a 3 for x.

$$x^2 - 2x + 3 = (3)^2 - 2(3) + 3 = 9 - 6 + 3 = 6.$$

The answer is (C).

Question 2.

The answer is (D).

Explanation for Question 2:

$$\begin{aligned} f(-4) &= ((-4) + 2)((-4) - 4) + 2(-4) = \\ &= (-2)(-8) + (-8) = 16 + (-8) = 8. \end{aligned}$$

The answer is (D).

Question 3.

The answer is (D).

Explanation for Question 3:

$$G(10) = 100(10) - 5 = 1,000 - 5 = 995.$$

The answer is (D).

Question 4.

The answer is (A).

Explanation for Question 4:

$$r(3) = 2(3) + 1 = 6 + 1 = 7$$

$$s(2) = 3(2) - 4 = 6 - 4 = 2$$

$$r(3) + s(2) = 7 + 2 = 9.$$

The answer is (A).

Studyguide for the SAT Skill Quiz A: Substitution

Question 5.

The answer is (C).

Explanation for Question 5:

First find $r(3)$. $r(3) = (3)^2 = 9$.

Substitute 9 for $r(3)$ in the function $p(r(3))$.

$$p(9) = 2(9) + 1 = 18 + 1 = 19.$$

The answer is (C).

Question 6.

The answer is (D).

Explanation for Question 6:

$$g(-3) = (-3)^2 + 2(-3) + 3 = 9 - 6 + 3 = 6. \text{ The answer is (D).}$$

Question 7.

The answer is (D).

Explanation for Question 7:

$$4(3)^2(2)^3 = 4(9)(8) = 36(8) = 288.$$

The answer is (D).

Question 8.

The answer is (E).

Explanation for Question 8:

$$((2) + (3)) ((2)^2 - (2)(3) + 4) = (5)(4 - 6 + 4) = (5)(2) = 10.$$

The answer is (E).

Question 9.

The answer is (D).

Explanation for Question 9:

$$y = \frac{|(-2)| - 2(-2) - (-2)^2}{2 + 4 - 4} = \frac{2 + 4 - 4}{6 - 4} = \frac{2}{2} = 1.$$

The answer is (D).

Question 10.

The answer is (B).

Explanation for Question 10:

$$F(1) = \frac{(1) + 3}{(1) - 2} = \frac{4}{-1} = -4.$$

The answer is (B).



Question 1

If $x = -5$, then $-x^2 - x - 9 = ?$

- (A) -39
- (B) -29
- (C) 6
- (D) 11
- (E) 21

Question 2

If $a = -4.5$, then $a^3 - 5a^2 + 15 = ?$

- (A) -582.375
- (B) -400.125
- (C) -177.375
- (D) 4.875
- (E) 430.125

Question 3

If $r = 2$, what is the value of $15r - \left(\frac{20}{r}\right)r$?

- (A) -1,470
- (B) -720
- (C) -170
- (D) -70
- (E) 10

Question 4

If $a = 2$, $b = -4$, and $c = 0.5$, then what is the value of $\frac{a(b - 6c)}{b^2c}$?

- (A) -1.75
- (B) -1.25
- (C) -0.25
- (D) 0.25
- (E) 1.25

Question 5

If $b = 12$ and $c = 13$, then $\sqrt{c^2 - b^2} = ?$

- (A) 1
- (B) 5
- (C) 18
- (D) 25
- (E) 313

Question 6

The formula $P = 15x - 1000$ gives the profit in dollars of a toy company when it sells x number of teddy bears. If the company sells 150 teddy bears, what is the company's profit in dollars?

- (A) -12,750
- (B) 66.67
- (C) 1,250
- (D) 2,250
- (E) 3,250

Question 7

The weekly profit function of Hal's Hamburger Hut is

$P(x) = 2x^2 - 150x - 2000$ where x is the number of hamburgers sold in a week. If 100 hamburgers are sold in a week, what is Hal's weekly profit in dollars?

- (A) 30
- (B) 3,000
- (C) 7,500
- (D) 23,000
- (E) 53,000

Question 8

If $y = 2x^2 - x$, which of the following values of x will result in the largest value of y ?

- (A) -2
- (B) -1
- (C) 0
- (D) 1
- (E) 2

Question 9

If $b = 9 - 2a + a^2$, then which of the following values of a results in the smallest value of b ?

- (A) -2
- (B) -1
- (C) 0
- (D) 1
- (E) 2

Question 10

Given that $m = 3$, $n = -8$, and $p = 12$, evaluate $m + 5(n - np)$.

- (A) -4,160
- (B) -517
- (C) 3
- (D) 443
- (E) 704

Studyguide for the SAT Skill Quiz B: Substitution



Hint for Question 1:

Be careful with the negatives. Remember the order of operations.

Hint for Question 2:

Be careful with the negatives. Remember the order of operations.

Hint for Question 3:

Substitute 2 for r . Remember the order of operations.

Hint for Question 4:

Be careful with the signs and with the order of operations.

Hint for Question 5:

Everything underneath the square root must be simplified first.

Hint for Question 6:

Don't let the fact that this is a word problem intimidate you!
This is a basic substitution problem.

Hint for Question 7:

Don't let the fact that this is a word problem intimidate you!
This is a basic substitution problem.

Hint for Question 8:

Try each answer choice.
Don't just assume that the largest value
of x will result in the largest value of y .

Hint for Question 9:

Try each answer choice.
Don't just assume that the smallest
value of a will result in the smallest value of b .

Hint for Question 10:

Be careful with the negatives. Remember the order of operations.



Question 1.

The answer is (B).

Explanation for Question 1:

Substitute -5 in place of x :

$$-(-5)^2 - (-5) - 9$$

Remember that exponents come before multiplication.

$$-(25) - (-5) - 9 =$$

$$-25 + 5 - 9 =$$

$$-29$$

The answer is (B).

Question 2.

The answer is (C).

Explanation for Question 2:

Substitute -4.5 in place of a :

$$(-4.5)^3 - 5(-4.5)^2 + 15$$

Remember that exponents come before multiplication.

$$-91.125 - 5(20.25) + 15$$

Multiplication comes next:

$$-91.125 - 101.25 + 15$$

Now add and subtract:

$$-177.375$$

The answer is (C).

Studyguide for the SAT Skill Quiz B: Substitution

Question 3.

The answer is (D).

Explanation for Question 3:

Substitute 2 in place of r:

$$15(2) - \left(\frac{20}{2}\right)^2$$

Simplify inside the parentheses:

$$15(2) - (10)^2$$

Exponents come next:

$$15(2) - 100$$

Multiplication:

$$30 - 100 = -70$$

The answer is (D).

Question 4.

The answer is (A).

Explanation for Question 4:

Substitute in the values for a, b, and c:

$$\frac{a(b - 6c)}{b^2c}$$

Start inside the parentheses:

$$\frac{2(-4 - 6(0.5))}{(-4)^2(0.5)}$$

Simplify the entire numerator and entire denominator. Divide last.

$$\frac{2(-4 - 3)}{16(0.5)} = \frac{-14}{8} = -1.75$$

The answer is (A).

Studyguide for the SAT Skill Quiz B: Substitution

Question 5.

The answer is (B).

Explanation for Question 5:

Replace b and c with their values:

$$\sqrt{c^2 - b^2} = \sqrt{13^2 - 12^2}$$

Treat what is under the square root as you would parentheses.

It must be simplified first.

$$\sqrt{169-144} = \sqrt{25} = 5$$

The answer is (B).

Question 6.

The answer is (C).

Explanation for Question 6:

$P = 15x - 1,000$ is a formula where x = number of teddy bears and P = profit.

When $x = 150$, $P = ?$

$$P = 15(150) - 1,000 = 2,250 - 1,000 = 1,250$$

The answer is (C).

Question 7.

The answer is (B).

Explanation for Question 7:

$P(x) = 2x^2 - 150x - 2000$ is a formula where x = number of hamburgers and P = profit. Note that $P(x)$ is function notation; it does not mean P times x !

$$\begin{aligned} P(100) &= 2(100)^2 - 150(100) - 2,000 \\ &= 2(10,000) - 150(100) - 2,000 \\ &= 20,000 - 15,000 - 2,000 \\ &= 3,000 \end{aligned}$$

The answer is (B).

Question 8.

The answer is (A).

Explanation for Question 8:

Try each answer choice.

$$(A) \quad y = 2(-2)^2 - (-2) = 2(4) + 2 = 10$$

$$(B) \quad y = 2(-1)^2 - (-1) = 2(1) + 1 = 3$$

$$(C) \quad y = 2(0)^2 - (0) = 0$$

$$(D) \quad y = 2(1)^2 - 1 = 2(1) - 1 = 1$$

$$(E) \quad y = 2(2)^2 - 2 = 2(4) - 2 = 6$$

The answer is (A).

Question 9.

The answer is (D).

Explanation for Question 9:

Try each answer choice.

$$(A) \quad b = 9 - 2(-2) + (-2)^2 = 9 + 4 + 4 = 17$$

$$(B) \quad b = 9 - 2(-1) + (-1)^2 = 9 + 2 + 1 = 12$$

$$(C) \quad b = 9 - 2(0) + (0)^2 = 9$$

$$(D) \quad b = 9 - 2(1) + 1^2 = 9 - 2 + 1 = 8$$

$$(E) \quad b = 9 - 2(2) + 2^2 = 9 - 4 + 4 = 9$$

The answer is (D).

Studyguide for the SAT Skill Quiz B: Substitution

Question 10.

The answer is (D).

Explanation for Question 10:

Substitute in the values of m, n, and p.

$$3 + 5(-8 - (-8)12)$$

Simplify inside the parentheses first:

$$3 + 5(-8 + 96)$$

$$3 + 5(88)$$

Multiply next, then add:

$$3 + 440 = 443$$

The answer is (D).



Question 1

If $a = -20$ and $b = -1$, then

$$\frac{2}{3}|a - 5b| = ?$$

- (A) 25.67
- (B) 16.67
- (C) 15.67
- (D) 15
- (E) 10

Question 2

If $r = 9$, $s = 5$, and $t = -1$, then what is the value of

$$r^2 - 2[s + r(5 - t)]?$$

- (A) -87
- (B) -59
- (C) -37
- (D) -31
- (E) -1

Question 3

If $x = 4$ and $y = -1$, then $y - xy^3 = ?$

- (A) 5
- (B) 3
- (C) -5
- (D) -13
- (E) -65

Question 4

If $a = \frac{1}{3}$, then $\frac{1}{a^2} = ?$

- (A) $\frac{1}{9}$
- (B) $\frac{1}{6}$
- (C) 6
- (D) 9
- (E) It is undefined.

Question 5

If $x = 5$, then evaluate $\frac{9 - x^2}{3 - x}$.

- (A) 8
- (B) $\frac{4}{3}$
- (C) $\frac{1}{3}$
- (D) -8
- (E) -12.5

Question 6

For which value of x is the expression $\frac{x}{x+1}$ undefined?

- (A) -2
- (B) -1
- (C) 0
- (D) 1
- (E) 2

Question 7

For what non-negative value of x is the expression

$\frac{x-1}{x^2-16}$ undefined?

- (A) 1
- (B) 4
- (C) 8
- (D) 16
- (E) 256

Question 8

Given that $f(x) = 3x - 9$ and $g(x) = \frac{1}{2}x + 5$,
what is the value of $f(0) - g(8)$?

- (A) -18
- (B) -9
- (C) -8
- (D) 0
- (E) 18

Question 9

Given that $p = \frac{1}{v} + 4$, what is the value of p when $v = \frac{1}{4}$?

- (A) -3.75
- (B) 0.125
- (C) 4.25
- (D) 8
- (E) 16

Question 10

If $x = 9$ and $y = 2$, then $3x^y = ?$

- (A) 14
- (B) 54
- (C) 84
- (D) 243
- (E) 729

Studyguide for the SAT Skill Quiz C: Substitution



Hint for Question 1:

Everything inside the absolute value must be simplified first.

Hint for Question 2:

Be careful with the negatives. Remember the order of operations.

Hint for Question 3:

Be careful with the negatives. Remember the order of operations.

Hint for Question 4:

To divide by a fraction, take the reciprocal and multiply.

Hint for Question 5:

Simplify the entire numerator and denominator before dividing.

Hint for Question 6:

A fraction is undefined when the denominator equals zero.

Hint for Question 7:

A fraction is undefined when the denominator equals zero.

Hint for Question 8:

Find $f(0)$ by replacing x with zero in the $f(x)$ function.

Find $g(8)$ by replacing x with 8 in the $g(x)$ function.

Hint for Question 9:

Don't forget that to divide by a fraction you take the reciprocal and multiply.

Hint for Question 10:

Remember that exponents come before multiplication in the order of operations.



Answers!



Question 1.

The answer is (E).

Explanation for Question 1:

Substitute in the values of a and b:

$$\frac{2}{3}|a - 5b| = \frac{2}{3}|-20 - 5(-1)|$$

Simplify inside the absolute value first.

$$\frac{2}{3}|-20 - (-5)| = \frac{2}{3}|-15|$$

Remember that the absolute value of a negative number is always positive.

$$\frac{2}{3}15 = 10$$

The answer is (E).

Studyguide for the SAT Skill Quiz C: Substitution

Question 2.

The answer is (C).

Explanation for Question 2:

Substitute in the values of r, s, and t:

$$9^2 - 2[5 + 9(5 - (-1))]$$

Simplify inside the parentheses first and then the brackets.

$$9^2 - 2[5 + 9(5 + 1)] =$$

$$9^2 - 2[5 + 9(6)] =$$

$$9^2 - 2(5 + 54) =$$

$$9^2 - 2(59)$$

The exponent comes next:

$$81 - 2(59)$$

Now multiply and then subtract last:

$$81 - 118 =$$
$$-37$$

The answer is (C).

Question 3.

The answer is (B).

Explanation for Question 3:

Substitute in the values of x and y:

$$-1 - 4(-1)^3$$

The exponent comes first:

$$-1 - 4(-1)$$

Now multiply and then add last:

$$-1 + 4 =$$
$$3$$

The answer is (B).

Studyguide for the SAT Skill Quiz C: Substitution

Question 4.

The answer is (D).

Explanation for Question 4:

Replace a with $\frac{1}{3}$:

To divide by $(\frac{1}{3})^2$, take the reciprocal and multiply.

$$\frac{1}{\frac{1}{3}} = 3$$

$$\text{and } 3^2 = 9$$

The answer is (D).

Question 5.

The answer is (A).

Explanation for Question 5:

Replace x with 5:

Simplify the numerator and denominator:

$$\frac{9 - 25}{3 - 5} = \frac{-16}{-2} = 8$$

The answer is (A).

Question 6.

The answer is (B).

Explanation for Question 6:

A fraction is undefined when the denominator equals zero. Look at the denominator and think about what will make it equal zero.

When $x = -1$:

$x + 1$ will equal 0 and the expression will be undefined.

The answer is (B).

Studyguide for the SAT Skill Quiz C: Substitution

Question 7.

The answer is (B).

Explanation for Question 7:

A fraction is undefined when the denominator equals zero. Look at the denominator and think about what will make it equal zero.

You can try every answer choice if you are not sure.

(A) denominator = $1 - 16 = -15$

(B) denominator = $16 - 16 = 0$ undefined

The answer is (B).

Question 8.

The answer is (A).

Explanation for Question 8:

First find $f(0)$ and $g(8)$.

$$f(0) = 3(0) - 9 = 0 - 9 = -9$$

$$g(8) = \frac{8}{2} + 5 = 4 + 5 = 9$$

Now subtract the two values.

$$f(0) - g(8) = -9 - 9 = -18$$

The answer is (A).

Question 9.

The answer is (D).

Explanation for Question 9:

Replace v with $1/4$:

$$p = \frac{1}{1/4} + 4$$

Take the reciprocal of $1/4$ and multiply:

$$p = \frac{1}{1/4} + 4 = 4 + 4 = 8$$

The answer is (D).

Studyguide for the SAT Skill Quiz C: Substitution

Question 10.

The answer is (D).

Explanation for Question 10:

Replace x with 9 and y with 2:

$$3(9)^2$$

Only the 9 is being squared.

$$3(81) = 243$$

The answer is (D).



FUNCTIONS

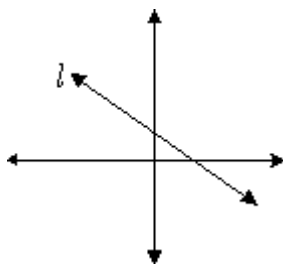
Knowing how to simplify functions is an important part of any math review. There are some concepts to review before getting into the definition of a function.

Definition of a relation: A relation is any set of ordered pairs. Ordered pairs can be expressed in a number of ways.

Examples of relations:

1. $\{(1,2), (2,3), (3,4)\}$
2. $\{(1,1), (1,3), (1,4)\}$
3. $\{(1,2), (2,2), (3,2), (4,2)\}$
4. $\{(x,y)|y = 2x + 3\}$
5. $\{(x,y)|y^2 = x\}$

You can even graph a relation on a coordinate system.



The graph of line l represents a relation.

Studyguide for the SAT Skill Review: Functions

In a relation, the set of first coordinates is called the Domain and the set of second coordinates is called the Range. Identify the domain (D) and range (R) for each of the above relations.

1. $\{(1,2), (2,3), (3,4)\}$ D = $\{1, 2, 3\}$, R = $\{2, 3, 4\}$
2. $\{(1,1), (1,3), (1,4)\}$ D = $\{1\}$, R = $\{1, 3, 4\}$
3. $\{(1,2), (2,2), (3,2), (4,2)\}$ D = $\{1, 2, 3, 4\}$, R = $\{2\}$
4. $\{(x,y) | y = 2x + 3\}$ D = $\{x | x \text{ is a member of } \mathbf{R}\}$,
R = $\{y | y \text{ is a member of } \mathbf{R}\}$
5. $\{(x,y) | y^2 = x\}$ D = $\{x | x \geq 0\}$,
R = $\{y | y \text{ is a member of } \mathbf{R}\}$

Where \mathbf{R} is the set of Real Numbers.

(Relation 4 is read, "The set of ordered pairs x, y such that y equals 2x plus 3."
The | represents the phrase "such that.")

Definition of a function: A function is a set of ordered pairs in which each element of the domain is assigned (or mapped to) only one element of the range.

Do you understand this definition? Look at the above relations. A function is a set of ordered pairs. Each element in the domain of a function can have only one range element assigned to it. Relations 1, 3 and 4 represent functions.

Relations 2 and 5 are not functions.

The reason relation 2 is not a function is because the domain element 1 is assigned to the range elements 1, 3, and 4 (more than one range element).

There are a number of ways to show example relation 5 is not a function.

Here is one.

If x equaled 25, then y would equal either 5 or -5. This would give an ordered pair solution to relation 5 of (25, 5) or (25, -5).

The domain element 25 would be assigned to more than one range element.

Studyguide for the SAT Skill Review: Functions

An easy way to express a function is to use the notation $F(x)$ or $G(x)$ or $H(x)$ ($f(x)$, $g(x)$, and $h(x)$ are also used). In these cases the function is F or G or H and the x represents the variable for the domain elements. The following is an example of a function:

$F(x) = 3x + 2$. The domain is x and the range is $3x + 2$.

This notation makes it easy to find ordered pairs that represent the function. For example, if $F(x) = 3x + 2$ then $F(4) = 3(4) + 2 = 14$.

You substitute 4 for x in the function. It is sometimes helpful to place parentheses around the number you are substituting for x . The value of $F(4) = 14$ and one ordered pair of the function F is $(4, 14)$.

You do not always need to have a numerical value to evaluate.

Example 1:

If $F(x) = 3x + 2$, find $F(a)$.

Solution:

To find $F(a)$ you substitute a for x in the function $F(x)$.

$$F(x) = 3x + 2$$

$$F(a) = 3a + 2. \text{ The value of } F(a) \text{ is } 3a + 2.$$

Example 2:

If $F(x) = 3x + 2$, find $F(a + 1)$.

Solution:

To find $F(a + 1)$ you substitute $a + 1$ for x in the function $F(x)$.

$$F(x) = 3x + 2$$

$$F(a + 1) = 3(a + 1) + 2 = 3a + 3 + 2 = 3a + 5.$$

$$F(a + 1) = 3a + 5$$

(Do you see how the parentheses help eliminate easy mistakes?)

Example 3:

If $F(x) = 3x + 2$, find $F(2x + 1)$.

Solution:

Don't let the x in $2x + 1$ confuse you.

Substitute $2x + 1$ for x in the function $F(x)$.

$$F(x) = 3x + 2$$

$$F(2x + 1) = 3(2x + 1) + 2 = 6x + 3 + 2 = 6x + 5.$$

$$F(2x + 1) = 6x + 5.$$

Composition of Functions

Composition of functions involves taking a function of a function. While it may seem confusing at the beginning it is not very difficult if you understood the last example.

Let $g(x) = x + 3$ and $h(x) = x^2 - 4$. Find $g(h(x))$.

To evaluate $g(h(x))$ begin with the innermost function $h(x)$.

Substitute in the expression for $h(x)$ to get $g(x^2 - 4)$.

Now evaluate $g(x^2 - 4)$.

$$g(x^2 - 4) = (x^2 - 4) + 3 = x^2 - 1.$$

$$g(h(x)) = x^2 - 1.$$

Please note that $g(h(x))$ is not an indicated multiplication. It is not g times h .

Let's reverse the functions.

Let $g(x) = x + 3$ and $h(x) = x^2 - 4$. Find $h(g(x))$.

$$h(g(x)) = h(x + 3) = (x + 3)^2 - 4 = x^2 + 6x + 9 - 4 = x^2 + 6x + 5.$$

$$h(g(x)) = x^2 + 6x + 5.$$

Direct and Inverse Variation

Direct Variation

A variable varies directly as another if it is in the form $y = ax$, where a is some constant.

For example, if $y = 4x$, then y is said to vary directly as x .

In this example the constant is 4.

Inverse Variation

A variable varies inversely as another if it is in the form $y = a/x$, where a is some constant.

For example, if $y = 3/x$, then y is said to vary inversely as x .

In this example the constant is 3.



Question 1

Given $f(x) = 3x + 4$, what is the value of $f(4)$?

- (A) 4
- (B) 7
- (C) 16
- (D) $12x + 16$
- (E) $12x + 4$

Question 2

Given $g(x) = x^2 - 2x + 5$, what is the value of $g(a)$?

- (A) a
- (B) $a^2 - 2a + 5$
- (C) $-a + 5$
- (D) $ax^2 - 2ax + 5a$
- (E) $ax^2 - 2ax + 5$

Question 3

Given $h(x) = x^2 + 3x - 7$, what is the value of $h(x + 1)$?

- (A) $x^2 + 5x - 3$
- (B) $x^2 + 3x - 6$
- (C) $x^2 + 3x - 3$
- (D) $x^2 + 5x - 5$
- (E) $x^2 + 3x - 5$

Question 4

Given $f(x) = x + 2$ and $g(x) = x^2$, what is the value of $f(g(x))$?

- (A) $x^3 + 2x^2$
- (B) $x + 2$
- (C) $x^2 + 2$
- (D) $x^2 + 4x + 4$
- (E) $x^2 + 4$

Question 5

Given $f(x) = x + 5$ and $g(x) = x^2 + 1$, what is the value of $g(f(x))$?

- (A) $x^2 + 26$
- (B) $x^2 + 6$
- (C) $x^2 + x + 6$
- (D) $x^2 + 10x + 26$
- (E) $x^3 + 5x^2 + x + 5$

Question 6

Given the function $f = \frac{3gf}{r}$, what is the value of r ?

- (A) $3gf^2$
- (B) $3g$
- (C) $\frac{1}{3g}$
- (D) $\frac{3}{f}$
- (E) $\frac{3f^2}{f}$

Question 7

Which of the following equations has y varying directly as x squared?

(A) $y = \frac{1}{x^2}$

(B) $y = 2x$

(C) $1 = x^2 y$

(D) $y = 3x^2$

(E) $y = \frac{5}{x^2}$

Question 8

Which of the following equations has y varying inversely as z cubed?

(A) $yz^3 = 2$

(B) $y = 3z^3$

(C) $3y = z^3$

(D) $4 = z^3 / y$

(E) $y^3 = 4/z$

Question 9

Which of the following has y varying directly as x cubed and inversely as z squared?

(A) $y = 4x^3 z^2$

(B) $x^3 y = 4z^2$

(C) $1 = \frac{3x^3}{yz^2}$

(D) $\frac{2}{z} = \frac{5y}{x^3}$

(E) $x^3 = \frac{5y}{2z^2}$

Question 10

If $g(x) = 2x^3 - x + 5$, then $g(2) =$

- (A) 6
- (B) 7
- (C) 11
- (D) 19
- (E) 20



Hint for Question 1:

To evaluate $f(4)$ substitute the 4 for the x in $f(x)$.

Hint for Question 2:

To evaluate $g(a)$ substitute a for x in the function $g(x) = x^2 - 2x + 5$.

Hint for Question 3:

To evaluate $h(x + 1)$ substitute $x + 1$ in for x in the function $h(x)$.

If you have problems with all of the x 's, evaluate $h(a + 1)$ first and then change all of the a 's to x 's.

Hint for Question 4:

Substitute x^2 for $g(x)$.

The expression will look like this $f(x^2)$.

Hint for Question 5:

Substitute $x + 5$ for $f(x)$ in $g(f(x))$.
You should be evaluating the function $g(x + 5)$.

Hint for Question 6:

Solve for r the same way you would solve for any variable.

Hint for Question 7:

What does it mean for a variable to "vary directly"? Be sure to solve for y before arriving at your conclusion.

Hint for Question 8:

Review what it means to "vary inversely" and look for the equation that fits that form. Remember to solve for y before reaching your conclusion.

Hint for Question 9:

An example of a variable that varies directly is in the form $y = ax$.
An example of a variable that varies inversely is in the form $y = a/z$.
When you combine the two, one variable is in the numerator and the other variable is in the denominator. Be sure you know where to place the variables x and z .
Remember to solve for y before arriving at your conclusion.

Hint for Question 10:

Substitute 2 for x .



Question 1.

The answer is (C).

Explanation for Question 1:

To evaluate this function substitute
4 for x in the function.

$$\begin{aligned}f(x) &= 3x + 4 \\f(4) &= 3(4) + 4 \\&= 12 + 4 \\&= 16\end{aligned}$$

The answer is (C).

Question 2.

The answer is (B).

Explanation for Question 2:

To answer this problem substitute
 a for x in the function

$$g(x) = x^2 - 2x + 5$$

$$g(a) = a^2 - 2a + 5$$

There is no further simplifying.

The answer is (B).

Question 3.

The answer is (A).

Explanation for Question 3:

To answer this problem substitute $x + 1$ for x in the function

$$h(x) = x^2 + 3x - 7 \text{ and then simplify the result.}$$

$$h(x + 1) = (x + 1)^2 + 3(x + 1) - 7 =$$

$$x^2 + 2x + 1 + 3x + 3 - 7 =$$

$$x^2 + 5x - 3$$

The answer is (A).

Question 4.

The answer is (C).

Explanation for Question 4:

To evaluate $f(g(x))$ correctly you substitute x^2 for $g(x)$

and then evaluate $f(x^2)$.

$$f(g(x)) = f(x^2) =$$

$$(x^2) + 2 = x^2 + 2$$

The answer is (C).

Question 5.

The answer is (D).

Explanation for Question 5:

To evaluate $g(f(x))$, first substitute $x + 5$ for $f(x)$.

$$g(x + 5) = (x + 5)^2 + 1$$

$$= x^2 + 10x + 25 + 1$$

$$= x^2 + 10x + 26$$

If you have difficulty working with all of the x 's, change $x + 5$ to $a + 5$. It may be simpler evaluating $g(a + 5)$.

The answer is (D).

Question 6.

The answer is (B).

Explanation for Question 6:

$$f = \frac{3gf}{r}$$

To solve for r you need to first remove r from the denominator. One way to do this is to multiply both sides of the equation by r.

$$rf = \frac{3gf}{r}(r)$$

$rf = 3gf$ Divide both sides of the equation by f.

$$\frac{rf}{f} = \frac{3gf}{f}$$

$r = 3g$ The answer is (B).

Question 7.

The answer is (D).

Explanation for Question 7:

A variable varies directly as another if it is in the form $y = az$, where a is some constant.

A variable will vary directly as the square

of x if $y = ax^2$.

The answer is (D).

Question 8.

The answer is (A).

Explanation for Question 8:

For y to vary inversely as z cubed the equation

must fit the form $y = a/z^3$ where a is some constant. To vary inversely z^3 must be in the denominator.

a represents some constant.

Make sure you solve all choices for y before making your decision.

Answer (A) must be changed to the equivalent

equation $y = 2/z^3$. This equation fits

the form $y = a/z^3$.

The answer is (A).

Question 9.

The answer is (C).

Explanation for Question 9:

When you solve for y in answer (C), you get $y = \frac{3x^3}{z^2}$.

In this equation, y varies

directly as x cubed and inversely as z squared.

The answer is (C).

Question 10.

The answer is (D).

Explanation for Question 10:

$$2x^3 - x + 5 = 2(2)^3 - 2 + 5 = 2(8) - 2 + 5 = 19$$

The answer is (D).

Math Lesson #7

Equations and Inequalities

- *Basic Linear Equations*
- *Special Linear Equations*
- *Rational Linear Equations*
- *Inequalities*
- *Factor and Expression*
- *Proportions*
- *Cancel Common Terms*
- *Quadratic Inequalities*

Triumph College Admissions

 **Skill Lesson**

 **Quiz Time!**

 **Hints!**

 **Answers!**

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BASIC LINEAR EQUATIONS

Basic algebra is an important skill to know for the SAT. In this lesson you will review the processes used to solve linear equations with one variable.

To Solve an Equation

What does it mean to solve an equation?

When you are given an equation such as $x - 7 = 12$ and you are asked to solve for x , you are trying to find the value of the variable that will make the equation true. One approach is to substitute numbers for x until one of the numbers works.

This technique is called backsolving on a multiple-choice test. Because you are given 5 answer choices you can substitute in one number at a time and find the one that makes the equation true.

Example 1:

If $x - 7 = 12$, what is the value for x ?

(A) 5 (B) 10 (C) 15 (D) 19 (E) 21

Solution:

When backsolving you should begin with answer choice (C). On the SAT the answer choices are written either from least to greatest or greatest to least. If you start with answer choice (C) and it is too big you know to next try a smaller answer choice. If answer choice (C) is too small then you next try a larger answer choice.

To use backsolving in solving example 1, first substitute in answer choice (C) (15) for x . Does $15 - 7 = 12$? No, $15 - 7 = 8$. Not true.

Because 8 is smaller than 12 you need to try a larger answer choice.

Try substituting 19 for x .

Does $19 - 7 = 12$? Yes, $19 - 7 = 12$. True. The answer is (D).

Studyguide for the SAT Skill Review: Basic Linear Equations

You can use backsolving in a number of applications. Sometimes it is faster than solving the problem "algebraically."

Another approach to solve equations is using algebra. The following is an algebra review.

Your objective is to find the value of the variable that makes the equation true. You can do this by transforming the equation. This means you can put the equation into another form.

One technique used to transform equations is to add or subtract the same value from both sides of the equation.

The equal sign in an equation indicates that both sides of the equation are equal in value, so whatever operation you perform on one side of the equation must be performed on the opposite side of the equation.

Rule 1. If $a = c$, then $a + d = c + d$.

This rule states that if a and c are equal in value, and if you add d to one side of the equation, then to keep both sides equal, you must add d to the other side of the equation.

Example 2:

If $x - 15 = 23$, then $x = ?$

Solution:

Get used to looking at sides of the equation. One side is " $x - 15$ " and the other side is "23." In reality you can add any number you wish to both sides of this equation. You could add 8 to both sides and get the resulting equation.

$$x - 15 + 8 = 23 + 8$$

$$x - 7 = 31.$$

You have transformed the equation! It looks different from the original equation but the solution for x has not changed. Keep in mind that the power you have to transform equations is the tool to solving for the variable. Why? To solve an equation you always want to transform it to the point where one side of the equation is only the variable. Your goal is to get x by itself.

Studyguide for the SAT Skill Review: Basic Linear Equations

Try example 2 again.

$x - 15 = 23$ What number would you add to " $x - 15$ " so only the " x " remained?

$x - 15 + 15 = 23 + 15$ That is correct. You would add 15. Now add it to both sides.

$x = 38$ The " x " is by itself. You have solved the equation.

That was a lot of work to review some basic algebra, but getting the variable by itself is the objective when solving an equation.

Rule 1. If $a = c$, then $a + d = c + d$.

You can add the same value to both sides of an equation.

Rule 2. If $a = c$, then $a - d = c - d$.

You can subtract the same value from both sides of an equation.

Rule 3. If $a = c$, then $ad = cd$.

You can multiply both sides of an equation by the same value.

Rule 4. If $a = c$, then $\frac{a}{d} = \frac{c}{d}$, $d \neq 0$

You can divide both sides of an equation by the same value.

Example 3:

If $x + 5 = 8$, what is the value of x ?

Solution:

$$x + 5 = 8$$

$$x + 5 - 5 = 8 - 5$$

$$x = 3$$

Always do the addition or subtraction transformations before doing the multiplication and division transformations.

Example 4:

If $3x = 132$, then x equals?

Solution:

$$3x = 132$$

$$\frac{3x}{3} = \frac{132}{3}$$

$$x = 44$$

Example 5:

If $\frac{x}{5} = 30$, what is the value of x ?

Solution:

$$\frac{x}{5} = 30$$

$$5 \times \frac{x}{5} = 5 \times 30$$

$x = 150$. This type of problem always seems the most difficult for students to solve.

Multiple-Step Problems

The next basic algebra type problems combine the use of rules 1 through 4. Remember to use the addition and subtraction transformations before the multiplication and division transformations.

Example 6:

Solve for y , if $3y - 23 = 43$.

Solution:

$$3y - 23 = 43$$

$$3y - 23 + 23 = 43 + 23$$

$$3y = 66$$

$$\frac{3y}{3} = \frac{66}{3}$$

$$y = 22$$

Example 7:

Solve for z, if $\frac{z}{6} + 12 = -5$.

Solution:

$$\frac{z}{6} + 12 = -5$$

$$\frac{z}{6} + 12 - 12 = -5 - 12$$

$$\frac{z}{6} = -17$$

$$6 \times \frac{z}{6} = 6 \times (-17)$$

$$z = -102$$

The only things that make these problems more difficult are increasing the number of steps. You may have to simplify one or both sides of the equation before solving, and/or you may have to get the variable on just one side of the equation.

Here is an example of needing to simplify both sides of the equation.

Example 8:

If $2(x - 8) = 3(10 - 2)$, what is the value of x?

Solution:

$$2(x - 8) = 3(10 - 2)$$

$$2x - 16 = 3(8)$$

$$2x - 16 = 24$$

$$2x - 16 + 16 = 24 + 16$$

$$2x = 40$$

$$\frac{2x}{2} = \frac{40}{2}$$

$$x = 20$$

Studyguide for the SAT Skill Review: Basic Linear Equations

This is an example of an equation with a variable on both sides.

Example 9:

If $5x - 10 = 7x + 34$, what is the value of x ?

Solution:

To get all variables on the same side of the equation you use the transformation rules. These rules work with whatever value you want to move.

$5x - 10 = 7x + 34$. For some reason, most students like the variable to be on the left side of the equation. It really doesn't matter and sometimes you won't make a mistake with positive and negative numbers if you keep the coefficient of the variable positive. Both solutions are shown.

Solutions:

$$5x - 10 = 7x + 34$$

$$5x - 10 - 7x = 7x + 34 - 7x$$

$$-2x - 10 = 34$$

$$-2x - 10 + 10 = 34 + 10$$

$$-2x = 44$$

$$\frac{-2x}{-2} = \frac{44}{-2}$$

$$x = -22$$

$$5x - 10 = 7x + 34$$

$$5x - 10 - 5x = 7x + 34 - 5x$$

$$-10 = 2x + 34$$

$$-10 - 34 = 2x + 34 - 34$$

$$-44 = 2x$$

$$\frac{-44}{2} = \frac{2x}{2}$$

$$-22 = x$$

You get the same answer, but in the second solution you eliminate a possible sign mistake.



Question 1

Solve for x , if $x + 27 = 34$.

- (A) 6
- (B) 7
- (C) 8
- (D) 51
- (E) 61

Question 2

Solve for x , if $2x - 15 = 37$.

- (A) 6
- (B) 11
- (C) 26
- (D) 52
- (E) 104

Question 3

Solve for x , if $\frac{x}{3} = 9$.

- (A) 3
- (B) 6
- (C) 9
- (D) 12
- (E) 27

Question 4

If $x + 2 = 11$, then $x + 5 = ?$

- (A) 9
- (B) 13
- (C) 14
- (D) 16
- (E) 18

Question 5

If $235 - x = 412$, then $x = ?$

- (A) -187
- (B) -177
- (C) 177
- (D) 187
- (E) 647

Question 6

For what value of x is $2(x + 3) - 4x = 12$?

- (A) -9
- (B) -3
- (C) 1
- (D) 3
- (E) 9

Question 7

Solve for x , if $5(x + 1) - 3(2x + 4) = 2(x + 7)$.

- (A) -7
- (B) $-2/3$
- (C) 1
- (D) 7
- (E) 21

Question 8

For what value of x is $43 + x = -22$ true?

- (A) -65
- (B) -21
- (C) 12
- (D) 21
- (E) 65

Question 9

Solve for x , if $7x - 3(x + 5) = 45$.

- (A) 3
- (B) 7.5
- (C) 10
- (D) 12.5
- (E) 15

Question 10

Solve for x , if $(2x + 3) - (2x + 1) = x$.

- (A) $-4/3$
- (B) $-2/3$
- (C) $4/3$
- (D) 2
- (E) 4



Hint for Question 1:

You want to isolate the variable.
Use opposite operations to move unwanted numbers away from the variable.

Hint for Question 2:

You want to isolate the variable. You need to use two opposite operations to move unwanted numbers away from the variable.
Move numbers that are added or subtracted first.

Hint for Question 3:

You want to isolate the variable.
The opposite operation of division is multiplication.

Hint for Question 4:

While you may solve for x to get this solution, think about trying to solve for $x + 5$ directly.
Remember you can add any number to both sides of an equation.

Hint for Question 5:

You want to isolate the variable.
Don't forget about the negative sign in front of the x .
Remember the opposite of a negative is a positive.

Hint for Question 6:

Before you try to isolate the variable, be sure to simplify the left side of the equation.
Don't forget about the distributive property, and how to add and divide positive and negative numbers.

Studyguide for the SAT Skill Quiz A: Basic Linear Equations

Hint for Question 7:

Before you try to isolate the variable, be sure to simplify both sides of the equation.
Then get all variables on the same side of the equation before isolating the variable.
Be careful when distributing the -3.

Hint for Question 8:

Isolate the variable and be careful of the negative signs.

Hint for Question 9:

Before you isolate the variable, be sure to simplify the left side of the equation.
Be careful when distributing the negative 3.

Hint for Question 10:

Simplify the left side of the equation before isolating the x .
Be careful of the negative sign in front of $(2x + 1)$.



Answers!

(A) (B) (C) (D) (E)



Question 1.

The answer is (B).

Explanation for Question 1:

You want to move the 27 to the other side of the equation.

The opposite operation to adding 27 is subtracting 27.

Write the following:

$$x + 27 = 34$$

$$x + 27 - 27 = 34 - 27$$

$$x = 7$$

The answer is (B).

Question 2.

The answer is (C).

Explanation for Question 2:

$$2x - 15 = 37$$

$$2x - 15 + 15 = 37 + 15$$

$$2x = 52$$

$$2x/2 = 52/2$$

$$x = 26$$
 The answer is (C).

Question 3.

The answer is (E).

Explanation for Question 3:

$$\frac{x}{3} = 9$$

$$\frac{x}{3}(3) = 9(3)$$

$$x = 27$$
 The answer is (E).

Question 4.

The answer is (C).

Explanation for Question 4:

To solve for $x + 5$ directly do the following:

$$x + 2 = 11$$

$$x + 2 + 3 = 11 + 3$$

$$x + 5 = 14 \text{ The answer is (C).}$$

The alternate way to solve the problem follows.

$$x + 2 = 11$$

$$x + 2 - 2 = 11 - 2$$

$$x = 9$$

$x + 5 = 9 + 5 = 14$ The answer is still (C),
but this way takes another step.

The first method is shorter.

Question 5.

The answer is (B).

Explanation for Question 5:

$$235 - x = 412$$

$$235 - x - 235 = 412 - 235$$

$$-x = 177$$

$$-(-x) = -(177)$$

$$x = -177 \text{ The answer is (B).}$$

Question 6.

The answer is (B).

Explanation for Question 6:

$$2(x + 3) - 4x = 12$$

$$2x + 6 - 4x = 12$$

$$-2x + 6 = 12$$

$$-2x + 6 - 6 = 12 - 6$$

$$-2x = 6$$

$$-2x/(-2) = 6/(-2)$$

$$x = -3 \text{ The answer is (B).}$$

Question 7.

The answer is (A).

Explanation for Question 7:

$$5(x + 1) - 3(2x + 4) = 2(x + 7)$$

$$5x + 5 - 6x - 12 = 2x + 14$$

$$-x - 7 = 2x + 14$$

$$-x - 7 + x = 2x + 14 + x$$

$$-7 = 3x + 14$$

$$-7 - 14 = 3x + 14 - 14$$

$$-21 = 3x$$

$$-21/3 = 3x/3$$

$$-7 = x \text{ The answer is (A).}$$

This solution kept the variable on the right side of the equation.

You could solve it with the variable on the left side of the equation.

If you have the variable on the left side of the equation, be sure your coefficient of x is a negative 3.

Question 8.

The answer is (A).

Explanation for Question 8:

$$43 + x = -22$$

$$43 + x - 43 = -22 - 43$$

$$x = -65 \text{ The answer is (A).}$$

Question 9.

The answer is (E).

Explanation for Question 9:

$$7x - 3(x + 5) = 45$$

$$7x - 3x - 15 = 45$$

$$4x - 15 = 45$$

$$4x - 15 + 15 = 45 + 15$$

$$4x = 60$$

$$4x/4 = 60/4$$

$$x = 15 \text{ The answer is (E).}$$

Studyguide for the SATSkill Quiz A: Basic Linear Equations

Question 10.

The answer is (D).

Explanation for Question 10:

$$(2x + 3) - (2x + 1) = x$$

$$2x + 3 - 2x - 1 = x$$

$$2 = x \text{ The answer is (D).}$$



ALGEBRA

You will need to know your basic algebra and develop the ability to "see" the solution to the problem from the given information. Many SAT problems appear to be much more difficult than they really are. When you are working on the skill quizzes for this section it is a good idea to check the hints and explanations even if you got the problem right. The explanations will guide you to time-saving solutions that will serve you well on test day.

Rule 1:

$a = b$ if, and only if, $a + c = b + c$

You can add or subtract the same quantity on both sides of an equation.

Example: If $x + 3 = 5$, what is the value of x ?
 $x + 3 - 3 = 5 - 3$
 $x = 2$

$a = b$ if, and only if, $a/c = b/c$

Rule 2:

You can multiply or divide the same quantity on both sides of an equation.

Example: If $3x = 24$, what is the value of x ?
 $3x/3 = 24/3$
 $x = 8$

Studyguide for the SAT Skill Review: Special Linear Equations

Often the questions will not be straightforward and ask you to solve for x or y . For example, they will ask you to solve for $x + 7$ or some other expression.

Example 1:

If $x + 3 = 5$, what is the value of $x + 7$?

Solution:

Most students are tempted to solve for x and then add 7 to their answer.

While the above method will work it is important to "see" that you are asked to solve for $x + 7$ not x . Look at this alternate solution for $x + 7$.

$$\begin{array}{r} x + 3 = 5 \\ + 4 \quad + 4 \\ \hline x + 7 = 9 \end{array}$$

By adding 4 to both sides of the equation you are able to find the value for $x + 7$ directly. This is a quicker approach than solving for x and then adding 7.

Example 2:

If $2x + 3y = 5$ and $3x + 4y = 6$, then what is the value of $5x + 7y$?

Solution:

This almost looks like you will need systems of equations to find x , then y , and substitute the answers into the expression $5x + 7y$. The question is not that hard. Always keep in mind what you want to answer. You want to find $5x + 7y$. If you set one equation above the other you will see a quick way to solve for $5x + 7y$.

$2x + 3y = 5$	If you add these two equations together
$\underline{3x + 4y = 6}$	the sum will be $5x + 7y$.
$5x + 7y = 11$	The solution to the question is 11.

On some problems, look for ways to get the answer without solving for a single variable. You may have to add, subtract, multiply, divide, or do some combination of operations with the given information to get the answer.



Question 1

If $3x + 6 = 16$, then $x - 1/3 = ?$

- (A) 2
- (B) 3
- (C) 6
- (D) 9
- (E) 12

Question 2

The product of 5, 6, and y is equal to the sum of $6y$ and which of the following?

- (A) $5y$
- (B) $24y$
- (C) 30
- (D) 12
- (E) 5

Question 3

If $3a + 4b = 12$ and $a = 5b/3$, then $b = ?$

- (A) $5/3$
- (B) $3/4$
- (C) 3
- (D) $4/3$
- (E) 4

Question 4

$$(5x^2 - 2x + 4) - (x + 3)(2x - 1) = ?$$

(A) $3x^2 - 7x + 7$

(B) $3x^2 + 7x + 7$

(C) $3x^2 - 7x + 1$

(D) $7x^2 - 7x + 7$

(E) $7x^2 + 3x + 1$

Question 5

$$\text{If } (n + 2)(7 - 4) = 15, \text{ then } n = ?$$

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

Question 6

$$\text{If } 3x + 2y = 7, \text{ what is the value of } 6x + 4y?$$

(A) 14

(B) 12

(C) $7/2$

(D) $7/3$

(E) 0

Question 7

If $x = 3ay^2$ and $y = 2r^3$, then x in terms of a and r is which of the following?

- (A) $12ar^6$
- (B) $6ar^6$
- (C) $12ar^5$
- (D) $6ar^5$
- (E) $5ar^5$

Question 8

If $(2x + 3)/3 = 11/3$, then $x = ?$

- (A) 8
- (B) 4
- (C) 2
- (D) $3/2$
- (E) 1

Question 9

If $x + 7 = 5$, what is the value of $x - 7$?

- (A) -9
- (B) -5
- (C) -2
- (D) 12
- (E) 19

Question 10

If $x + 2 = 8$, what is the value of $(x + 3)^2$?

- (A) 81
- (B) 49
- (C) 36
- (D) 16
- (E) 9



Hint for Question 1:

Solve for x and then subtract $1/3$.

Hint for Question 2:

Rewrite the words into algebra. Product means to multiply and sum means to add.

Hint for Question 3:

Substitute for a and then solve for b .

Hint for Question 4:

Multiply the binomials. Don't forget about FOIL (see Factor the Expression lesson) and don't miss the negative sign in front of the product.

Hint for Question 5:

Simplify $7 - 4$, distribute, and then solve.

Hint for Question 6:

This is easier than it looks. What can you do to $3x + 2y$ to get $6x + 4y$?

Hint for Question 7:

Substitute and then solve for x .

Hint for Question 8:

Remember how to cross-multiply? That's one way to solve for x .

Hint for Question 9:

One way is to solve for x and then subtract 7. There's a faster way.

Hint for Question 10:

You could solve for x , add 3, and then square the result.
There's a faster way for this one also.



Question 1.

The answer is (B).

Explanation for Question 1:

Solve for x.

$$3x + 6 = 16$$

$$3x = 10$$

$$x = 10/3$$

$$x - 1/3 = 10/3 - 1/3 = 9/3 = 3$$

The answer is (B).

Question 2.

The answer is (B).

Explanation for Question 2:

The question can be written as $5 \times 6 \times y = 6y + ?$

$$30y = 6y + ?$$

$$24y = ?$$

The answer is (B).

Question 3.

The answer is (D).

Explanation for Question 3:

Substitute $5b/3$ for a into the first equation.

$$3(5b/3) + 4b = 12$$

$$5b + 4b = 12$$

$$9b = 12$$

$$b = 12/9 = 4/3$$

The answer is (D).

Question 4.

The answer is (A).

Explanation for Question 4:

FOIL stands for First, Outer, Inner, Last. It is a way to multiply binomials.

Check the Skill Lesson Factor the Expression.

$$(x + 3)(2x - 1) = x \times 2x + x \times (-1) + 3 \times 2x + 3 \times (-1) = 2x^2 - x + 6x - 3 = 2x^2 + 5x - 3$$

Substituting for $(x + 3)(2x - 1)$ the problem becomes the following:

$$(5x^2 - 2x + 4) - (2x^2 + 5x - 3) = \text{(Be careful distributing the negative.)}$$

$$5x^2 - 2x + 4 - 2x^2 - 5x + 3 =$$

$$3x^2 - 7x + 7$$

The answer is (A).

Question 5.

The answer is (C).

Explanation for Question 5:

$7 - 4 = 3$. The problem is then the following:

$$(n + 2) \times 3 = 15$$

$$3n + 6 = 15$$

$$3n = 9$$

$$n = 3$$

The answer is (C).

Question 6.

The answer is (A).

Explanation for Question 6:

This is a common type of SAT problem. You have to "see" that $6x + 4y$ is $2(3x + 2y)$.

Realize you are not solving for x and y individually, but for the expression $6x + 4y$.

$$3x + 2y = 7$$

$$2(3x + 2y) = 2(7)$$

$$6x + 4y = 14$$

The answer is (A).

Question 7.

The answer is (A).

Explanation for Question 7:

Substitute $2r^3$ for y . $x = 3a(2r^3)^2$

First, simplify the power $x = 3a(4r^6)$ Remember to multiply exponents.

Second, multiply what you can. $x = 12ar^6$

The answer is (A).

Question 8.

The answer is (B).

Explanation for Question 8:

If you cross-multiply then you solve the following equation.

$$3(2x + 3) = 11(3)$$

$$6x + 9 = 33$$

$$6x = 24$$

$$x = 4$$

The answer is (B).

A faster way is to note if the denominators are equal in an equation, then the numerators must also be equal. You can then solve this equation.

$$2x + 3 = 11$$

$$2x = 8$$

$$x = 4$$

Question 9.

The answer is (A).

Explanation for Question 9:

One way is to solve for x.

$$x + 7 = 5$$

$$x = -2$$

$$\text{Then } x - 7 = -2 - 7 = -9$$

The answer is (A).

Another, faster way to solve for x?

The question asks you to find the value of the expression $x - 7$. Think about this approach.

$$x + 7 = 5$$

$$\underline{-14 \quad -14}$$

$$x - 7 = -9 \quad . \quad \text{This is faster. Always keep in mind what you are asked to find.}$$

Question 10.

The answer is (A).

Explanation for Question 10:

One way is to solve for x.

$$x + 2 = 8$$

$$x = 6$$

$$x + 3 = 9$$

$$(x + 3)^2 = 9^2 = 81$$

The faster way.

$$x + 2 = 8$$

$$\underline{+1 \quad +1}$$

$$x + 3 = 9$$

$$9^2 = 81$$

The answer is (A).



Question 1

If $a + b + c = 352$ and $a = -23$, what is the value of $b + c$?

(A) 329

(B) 375

(C) -329

(D) -375

(E) 319

Question 2

The product of 15, 20, and x is equal to the sum of $20x$ and which of the following?

(A) 15

(B) 280

(C) 300

(D) $15x$

(E) $280x$

Question 3

If $5x + 2y = 6$ and $3x - y = 10$, what is the value of $8x + y$?

(A) 8

(B) 4

(C) 16

(D) $5/3$

(E) 2

Question 4

If $y = 8 + x$ and $3y = 3 + 4x$, what is the value of y ?

- (A) -29
- (B) -5
- (C) 5
- (D) $35/4$
- (E) 29

Question 5

If $x = \frac{a^2}{b^3}$ and $b \neq 0$, then $\frac{1}{a^2} = ?$

- (A) xb^3
- (B) $x - b^3$
- (C) x/b^3
- (D) $1/xb^3$
- (E) b^3/x

Question 6

If $8x + 4y = 7$ and $5x + 2y = 3$, what is the value of $3x + 2y$?

- (A) 10
- (B) $\frac{7}{3}$
- (C) 4
- (D) -4
- (E) $\frac{3}{7}$

Question 7

If $a = 3b$ and $b = 6c$ and $3c = d$, then $\frac{a}{d} = ?$

- (A) $1/6$
- (B) $1/3$
- (C) 1
- (D) 3
- (E) 6

Question 8

If $x \times \frac{1}{y} = 1$, which of the following is NOT necessarily true?

- (A) $x^2 = y^2$
- (B) $x^2 = 1/y^2$
- (C) $x^2 + y^2 = 2xy$
- (D) $x = y$
- (E) $x + y = 2y$

Question 9

If $3x + 4y = 12$ and $x + 2y = 2$, what is the value of $2x + 3y$?

- (A) 7
- (B) 12
- (C) 15
- (D) 24
- (E) 30

Grid-in your answer here:

Question 10

If a and b are positive integers and $2a + b = 13$, what is the sum of all possible values of a?

	/	/	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

**Hint for Question 1:**

Substitute for a and solve for the expression $b + c$.

Hint for Question 2:

Rewrite as an equation and solve for the missing term.

Hint for Question 3:

You are not asked to solve for x or y . Place one equation above the other, lining up the variables. Can you see a way to get $8x + y$?

Hint for Question 4:

Solve the first equation for x and substitute this expression for x in the second equation. Solve for y .

Hint for Question 5:

First solve for a^2 , then solve for $1/a^2$.

Hint for Question 6:

You are not asked to solve for x or y . Place one equation above the other, lining up the variables. Can you see a way to get $3x + 2y$?

Hint for Question 7:

Try to get an equation with a 's and d 's in it. Substitution may help.

Hint for Question 8:

Rewrite the equation and see if you can duplicate each answer choice.

Hint for Question 9:

This may be more difficult to see. You are not asked to solve for x or y . Place one equation above the other, lining up the variables. You may not be able to see $3x + 2y$ but can you find a multiple of $3x + 2y$?

Hint for Question 10:

a is a positive integer. What are all of a 's possible values?



Question 1.

The answer is (B).

Explanation for Question 1:

Substitute -23 for a.

$$-23 + b + c = 352$$

$$\begin{array}{r} +23 \quad \quad +23 \\ \hline \end{array}$$

$$b + c = 375$$

The answer is (B).

Question 2.

The answer is (E).

Explanation for Question 2:

You need to solve the following equation.

$$15 \times 20 \times x = 20x + ?$$

$$300x = 20x + ?$$

$$280x = ?$$

The answer is (E).

Question 3.

The answer is (C).

Explanation for Question 3:

Use the hint to write the following:

$$5x + 2y = 6$$

$$\underline{3x - y = 10}$$

$$8x + y = 16 \text{ Add the equations together.}$$

The answer is (C).

Question 4.

The answer is (E).

Explanation for Question 4:

Solve for x in the first equation.

$$y = 8 + x$$

$$y - 8 = x$$

Substitute for x in the second equation.

$$3y = 3 + 4x$$

$$3y = 3 + 4(y - 8)$$

$$3y = 3 + 4y - 32$$

$$3y = 4y - 29$$

$$-y = -29$$

$$y = 29$$

The answer is (E).

Question 5.

The answer is (D).

Explanation for Question 5:

Solve for a^2 first.

$$x = \frac{a^2}{b^3} \quad \text{Multiply both sides by } b^3$$

$$xb^3 = a^2$$

Take the reciprocal of both sides of the equation to find $\frac{1}{a^2}$.

$$\frac{1}{xb^3} = \frac{1}{a^2}$$

The answer is (D).

Question 6.

The answer is (C).

Explanation for Question 6:

Use the hint to write the following:

$$8x + 4y = 7$$

$$\underline{5x + 2y = 3} \quad \text{Subtract the bottom equation from the top equation.}$$

$$3x + 2y = 4$$

The answer is (C).

Studyguide for the SAT Skill Quiz B: Special Linear Equations

Question 7.

The answer is (E).

Explanation for Question 7:

Substitute: $a = 3b$ and $b = 6c$, so $a = 3(6c)$

$$a = 18c$$

$3c = d$, so $6(3c) = 6d$

$$18c = 6d$$

$a = 18c$ and $18c = 6d$, so $a = 6d$ To solve for $\frac{a}{d}$ divide both sides by d .

$$\frac{a}{d} = 6 \text{ The answer is (E).}$$

Question 8.

The answer is (B).

Explanation for Question 8:

The given equation becomes $x = y$.

Check each answer choice. Use a number for x and y . If $x = 3$, then $y = 3$.

(A) If $x = y$, does $x^2 = y^2$? $9 = 9$ Yes.

(B) If $x = y$, does $x^2 = 1/y^2$? $9 = 1/9$ No. You can stop here. Let's do the rest for practice.

(C) If $x = y$, does $x^2 + y^2 = 2xy$? $9 + 9 = 2(3)(3)$, so $18 = 18$ Yes.

(D) If $x = y$, does $x = y$? Of course!

(E) If $x = y$, does $x + y = 2y$? $3 + 3 = 2(3)$ $6 = 6$ Yes.

The answer is (B).

Question 9.

The answer is (A).

Explanation for Question 9:

Use the hint to write the following:

$$3x + 4y = 12$$

$$\underline{x + 2y = 2}$$

$$4x + 6y = 14$$

You are looking for the value of the expression $2x + 3y$. $2x + 3y$ is $1/2$ of $4x + 6y$.

$$\frac{4x + 6y}{2} = \frac{14}{2}$$

$$2x + 3y = 7$$

The answer is (A).

Question 10.

The answer is 21.

Explanation for Question 10:

a and b are positive integers and $2a + b = 13$. List the possible values for a and b.

a	b
1	11
2	9
3	7
4	5
5	3
6	1

21 = the sum of the a column.

The answer is 21.



Question 1

If $xy = 8$ and $yz = 2$, what is the value of x/z ?

- (A) $1/4$
- (B) 1
- (C) 4
- (D) 8
- (E) 16

Question 2

If $3x + 5y = 5$ and $x + 3y = 2$, what is the value of $2x + 2y$?

- (A) 2
- (B) $5/2$
- (C) 3
- (D) 5
- (E) 7

Question 3

If $3x + 5y = 5$ and $x + 3y = 2$, what is the value of $4x + 8y$?

- (A) 2
- (B) $5/2$
- (C) 3
- (D) 5
- (E) 7

Question 4

If $3x + 5 = 4$, what is the value of $3x - 2$?

- (A) -3
- (B) -1
- (C) 2
- (D) 9
- (E) 10

Question 5

If $P = 3ay^2$, what is the value of a in terms of P and y ?

- (A) $3Py$
- (B) $3Py^2$
- (C) $3P/y^2$
- (D) $P/3y^2$
- (E) $Py^2/3$

Question 6

If $3x = 5$, then what is the value of $9x^2$?

- (A) $81/25$
- (B) 9
- (C) 25
- (D) $225/3$
- (E) 81

Question 7

If $5x - 2y = 12$, what is the value of $5x/2 - y$?

- (A) 4
- (B) 6
- (C) 10
- (D) 14
- (E) 16

Question 8

If $x + y = 3$ and $x + 2y = 4$, what is the value of $x^2 + 3xy + 2y^2$?

- (A) -1
- (B) 1
- (C) 7
- (D) 12
- (E) 34

Question 9

If $x + z = 3$ and $y + z = 5$, what is the value of $x - y$?

- (A) -2
- (B) $5/3$
- (C) 2
- (D) 8
- (E) 15

Question 10

If $xy = 4$, $yp = 2$, $xp = 2$, and $x > 0$, what is the value of xyp ?

- (A) 4
- (B) 8
- (C) 12
- (D) 16
- (E) 20

Studyguide for the SAT Skill Quiz C: Special Linear Equations



Hint for Question 1:

Divide one equation by the other.

Hint for Question 2:

You are not asked to solve for x or y . Place one equation above the other, lining up the variables. Can you see a way to get $2x + 2y$?

Hint for Question 3:

You are not asked to solve for x or y . Place one equation above the other, lining up the variables. Can you see a way to get $4x + 7y$?

Hint for Question 4:

Remember you are to find the value of the expression $3x - 2$.
What do you subtract from $3x + 5$ to get $3x - 2$?

Hint for Question 5:

Solve the equation for a .

Hint for Question 6:

What do you do to $3x$ to get $9x^2$?

Hint for Question 7:

What do you do to $5x - 2y$ to get $\frac{5x}{2} - y$?

Hint for Question 8:

Can you see the relationship between $x + y$, $x + 2y$, and $x^2 + 3xy + 2y^2$?

Hint for Question 9:

You are not asked to solve for x or y . Place one equation above the other, lining up the variables. Can you see a way to get $x - y$?

Hint for Question 10:

Look closely at xy , yp , and xp . Can you think of an operation to perform with these variables to get some type of expression with an xyp ?



Question 1.

The answer is (C).

Explanation for Question 1:

Divide $xy = 8$ by $yz = 2$. $\frac{xy}{yz} = \frac{8}{2}$, so $\frac{x}{z} = 4$

The answer is (C).

Question 2.

The answer is (C).

Explanation for Question 2:

Use the hint.

$$3x + 5y = 5$$

$x + 3y = 2$ Subtract the bottom equation from the top equation.

$$2x + 2y = 3$$

The answer is (C).

Question 3.

The answer is (E).

Explanation for Question 3:

Use the hint.

$$3x + 5y = 5$$

$x + 3y = 2$ Add the bottom equation to the top equation.

$$4x + 8y = 7$$

The answer is (E).

Question 4.

The answer is (A).

Explanation for Question 4:

To solve directly:

$$3x + 5 = 4$$

$$\quad -7 \quad -7$$

$$3x - 2 = -3$$

The answer is (A).

Question 5.

The answer is (D).

Explanation for Question 5:

In order to solve for a, divide both sides by 3 and y^2 .

$$\frac{P}{3y^2} = \frac{3ay^2}{3y^2}$$

$$\frac{P}{3y^2} = a$$

The answer is (D).

Question 6.

The answer is (C).

Explanation for Question 6:

$$9x^2 = (3x)^2 = 5^2 = 25$$

The answer is (C).

Question 7.

The answer is (B).

Explanation for Question 7:

Divide both sides of the equation by 2.

$$\frac{5x}{2} - \frac{2y}{2} = \frac{12}{2}$$

$$\frac{5x}{2} - y = 6$$

The answer is (B).

Question 8.

The answer is (D).

Explanation for Question 8:

If you multiply the two binomials you will get $x^2 + 3xy + 2y^2$
 $(x + y)(x + 2y) = 3(4)$

$$x^2 + xy + 2xy + 2y^2 = 12$$

$$x^2 + 3xy + 2y^2 = 12$$

The answer is (D).

Studyguide for the SAT Skill Quiz C: Special Linear Equations

Question 9.

The answer is (A).

Explanation for Question 9:

$$x + z = 3$$

$$\underline{y + z = 5} \quad \text{Subtract the bottom equation from the top equation.}$$

$$x - y = -2$$

The answer is (A).

Question 10.

The answer is (A).

Explanation for Question 10:

Multiply all of the variables. $xy(yp)(xp) = 4(2)(2)$

$$x^2 y^2 p^2 = 16. \quad \text{Take the square root of both sides.}$$

$$xyp = 4$$

The answer is (A).



RATIONAL LINEAR EQUATIONS

Some equations have the variable in the denominator. This skill lesson concentrates on ways to solve equations involving fractions.

The simplest equations involving fractions are those with rational coefficients. Look at the following example.

Example 1:

Solve for x , given $\frac{x}{2} + \frac{3}{5} = \frac{7}{2}$.

Solution:

This equation has numbers in the denominators.

To eliminate the denominators, first find the least common denominator (LCD) for all denominators. For this problem the LCD is 10.

You next multiply both sides of the equation by the LCD.

(This means you multiply each term by the LCD.)

$$10\left(\frac{x}{2} + \frac{3}{5}\right) = 10\left(\frac{7}{2}\right)$$

$$10\left(\frac{x}{2}\right) + 10\left(\frac{3}{5}\right) = 10\left(\frac{7}{2}\right)$$

$$5x + 6 = 35$$

$$5x = 29$$

$$x = \frac{29}{5}$$

This is the basis for solving all equations with fractions.

You first find the LCD and then multiply both sides of the equation by the LCD in order to eliminate all of the denominators.

You are left with an easier equation to solve.

Here is an equation with variables.

The process is the same as when you have only numbers.

Example 2:

Solve for x , given $\frac{3}{x} + \frac{2}{x+1} = \frac{1}{x}$

Solution:

The LCD for this problem is $x(x + 1)$.

The LCD must be divisible by both denominators x and $x + 1$.

Please note that x is not a factor of $x + 1$.

Multiply both sides of the equation by $x(x + 1)$.

$$x(x + 1)\left(\frac{3}{x} + \frac{2}{x + 1}\right) = x(x + 1)\frac{1}{x}$$

$$x(x + 1)\frac{3}{x} + x(x + 1)\frac{2}{x + 1} = x(x + 1)\frac{1}{x}$$

$$(x + 1)3 + 2x = x + 1$$

$$3x + 3 + 2x = x + 1$$

$$5x + 3 = x + 1$$

$$4x + 3 = 1$$

$$4x = -2$$

$$x = -2/4 = -1/2$$

There is one type of fraction that you do not have to bother with finding the LCD. This occurs when you have only one fraction on each side of the equation. To solve this type of equation you can cross-multiply the denominators. Look at the following example.

Example 3:

Solve for y , given $\frac{2y + 3}{y - 5} = \frac{4}{3}$.

Solution:

When you cross-multiply you multiply each denominator by the opposite numerator.

$$\frac{2y + 3}{y - 5} = \frac{4}{3}$$

$$3(2y + 3) = 4(y - 5)$$

$$6y + 9 = 4y - 20$$

$$2y = -29$$

$$y = -29/2$$

One final example looks like it is going to be a quadratic equation but as you simplify the problem the equation becomes a linear equation.

Example 4:

Solve for z , given $(z - 4)(z + 5) = (z + 6)(z + 3)$.

Solution:

$$(z - 4)(z + 5) = (z + 6)(z + 3)$$

$$z^2 + z - 20 = z^2 + 9z + 18$$

$$-8z - 20 = 18$$

$$-8z = 38$$

$$z = -38/8$$

$$z = -19/4$$

The z^2 's subtract and do not enter in the solution of the problem.



Question 1

Solve for x, given $\frac{x}{x+1} = \frac{4}{5}$.

- (A) -4
- (B) -1
- (C) 1
- (D) 4
- (E) 5

Question 2

Solve for x, given $\frac{3}{x} + \frac{5}{x} = \frac{1}{3}$.

- (A) 3
- (B) 8
- (C) 14
- (D) 18
- (E) 24

Question 3

Solve for x, given $\frac{2x+1}{3x+1} = \frac{4}{3}$.

- (A) -6
- (B) -1/6
- (C) 0
- (D) 1/6
- (E) 6

Question 4

Find the solution for x, given $x(y+3) - 2 = x(y+1)$.

- (A) y
- (B) -2
- (C) 1/y
- (D) 1/3
- (E) 1

Question 5

Solve for x , given $(x + 1)(x - 5) = (x + 2)(x - 7)$.

- (A) -14
- (B) -9
- (C) -5
- (D) 0
- (E) No solution

Question 6

Solve for x , given $\frac{x+3}{x-2} = \frac{5}{4}$.

- (A) -22
- (B) 1
- (C) 2
- (D) 14
- (E) 22

Question 7

Solve for x , given $\frac{3}{x+1} + \frac{2}{x} = \frac{1}{x}$.

- (A) $-\frac{3}{4}$
- (B) $-\frac{1}{4}$
- (C) $-\frac{1}{6}$
- (D) 0
- (E) 5

Question 8

Solve for y , given $\frac{4y+3}{2y-1} = \frac{3}{5}$.

- (A) $-\frac{9}{7}$
- (B) $-\frac{9}{14}$
- (C) $\frac{9}{14}$
- (D) $\frac{9}{7}$
- (E) 18

Question 9

Solve for z , given $\frac{3}{z} - \frac{1}{5} = \frac{1}{2}$.

- (A) -10
- (B) $-\frac{30}{7}$
- (C) 3
- (D) $\frac{30}{7}$
- (E) 10

Question 10

Solve for x , given $x(x - 3) = (x + 3)(x - 4)$.

- (A) -6
- (B) -4
- (C) 0
- (D) 4
- (E) 6



Hint for Question 1:

To solve this problem try cross-multiplying.

Hint for Question 2:

Find the common denominator for all three fractions.
Multiply both sides of the equation
by the common denominator.
The common denominator is $3x$.

Hint for Question 3:

You can solve this equation by cross-multiplying.
Don't forget the distributive law.

Hint for Question 4:

Simplify and get all variables on the
same side of the equation.

Hint for Question 5:

Simplify both sides and get all of the
variables on the same side of the equation.
Remember how to do FOIL.

Hint for Question 6:

You can solve this equation by cross-multiplying.

Hint for Question 7:

Multiply both sides of the equation by the common denominator.
The common denominator is $x(x + 1)$.

Hint for Question 8:

Hint:

You can solve this equation by cross-multiplying.

Hint for Question 9:

Multiply both sides of the equation by the common denominator.
The common denominator has factors of z , 5, and 2.

Hint for Question 10:

Simplify both sides of the equation and
get all of the variables on one side.
Don't forget about the distributive law
and FOIL.



Question 1.

The answer is (D).

Explanation for Question 1:

Cross-multiply to solve this problem.

$$\frac{x}{x+1} = \frac{4}{5}$$

$$5x = 4(x+1)$$

$$5x = 4x + 4$$

$$x = 4 \text{ The answer is (D).}$$

You could also backsolve to find the answer.

Read the math strategies section to learn the backsolving technique.

Question 2.

The answer is (E).

Explanation for Question 2:

Multiply both sides of the equation by the common denominator.

The common denominator is $3x$.

$$3x\left(\frac{3}{x} + \frac{5}{x}\right) = 3x\frac{1}{3}$$

Distribute the $3x$ on the left side of the equation.

$$3x\frac{3}{x} + 3x\frac{5}{x} = 3x\frac{1}{3}$$

Cancel the appropriate numerators and denominators.

$$9 + 15 = x$$

$$24 = x \text{ The answer is (E).}$$

Question 3.

The answer is (B).

Explanation for Question 3:

$$\frac{2x+1}{3x+1} = \frac{4}{3}$$

$$3(2x+1) = 4(3x+1)$$

$$6x+3 = 12x+4$$

$$-6x = 1$$

$$x = -1/6. \text{ The answer is (B).}$$

Question 4.

The answer is (E).

Explanation for Question 4:

$$x(y+3) - 2 = x(y+1)$$

$$xy + 3x - 2 = xy + x$$

$$2x - 2 = 0 \text{ The } xy \text{ terms subtract.}$$

$$2x = 2$$

$$x = 1 \text{ The answer is (E).}$$

Question 5.

The answer is (B).

Explanation for Question 5:

$$(x+1)(x-5) = (x+2)(x-7)$$

$$x^2 - 5x + x - 5 = x^2 - 7x + 2x - 14$$

$$x^2 - 4x - 5 = x^2 - 5x - 14$$

$$x = -9. \text{ The answer is (B).}$$

Studyguide for the SAT Skill Quiz A: Rational Linear Equations

Question 6.

The answer is (E).

Explanation for Question 6:

$$\frac{x+3}{x-2} = \frac{5}{4}$$

$$4(x+3) = 5(x-2)$$

$$4x + 12 = 5x - 10$$

$$-x = -22$$

$$x = 22. \text{ The answer is (E).}$$

Question 7.

The answer is (B).

Explanation for Question 7:

Multiply both sides of the equation by the common denominator. The common denominator must include all of the factors of each denominator. x is not a factor of $x + 1$. This is a common mistake. The common denominator has factors of x and $x + 1$. Hence the common denominator is $x(x + 1)$.

$$\frac{3}{x+1} + \frac{2}{x} = \frac{1}{x}$$

$$x(x+1)\left(\frac{3}{x+1} + \frac{2}{x}\right) = x(x+1)\frac{1}{x}$$

Distribute the $x(x+1)$ on the left side.

$$x(x+1)\frac{3}{x+1} + x(x+1)\frac{2}{x} = x(x+1)\frac{1}{x}$$

Cancel the denominators and numerators.

$$3x + (x+1)2 = x + 1$$

$$3x + 2x + 2 = x + 1$$

$$5x + 2 = x + 1$$

$$4x = -1$$

$$x = -1/4. \text{ The answer is (B).}$$

Studyguide for the SAT Skill Quiz A: Rational Linear Equations

Question 8.

The answer is (A).

Explanation for Question 8:

$$\frac{4y+3}{2y-1} = \frac{3}{5}$$

$$5(4y+3) = 3(2y-1)$$

$$20y+15 = 6y-3$$

$$14y = -18$$

$$y = -18/14 = -9/7. \text{ The answer is (A).}$$

Question 9.

The answer is (D).

Explanation for Question 9:

The common denominator is $10z$. Multiply both sides of the equation by $10z$.

$$\frac{3}{z} - \frac{1}{5} = \frac{1}{2}$$

$$10z\left(\frac{3}{z} - \frac{1}{5}\right) = 10z\frac{1}{2}$$

Distribute $10z$ through the left side of the equation.

$$10z\frac{3}{z} - 10z\frac{1}{5} = 10z\frac{1}{2}$$

Cancel the numerators and denominators.

$$30 - 2z = 5z$$

$$30 = 7z$$

$$30/7 = z. \text{ The answer is (D).}$$

Question 10.

The answer is (E).

Explanation for Question 10:

$$x(x-3) = (x+3)(x-4)$$

$$x^2 - 3x = x^2 - 4x + 3x - 12$$

$$x^2 - 3x = x^2 - x - 12$$

$$-2x = -12$$

$$x = 6. \text{ The answer is (E).}$$



Question 1

If $5 - (x + 6) = 8$, then $x = ?$

- (A) -9
- (B) -7.6
- (C) -3
- (D) 3
- (E) 9

Question 2

If $\frac{x+1}{5} = \frac{x}{3}$, then $x = ?$

- (A) $-\frac{15}{2}$
- (B) $-\frac{5}{2}$
- (C) 0
- (D) $\frac{1}{2}$
- (E) $\frac{3}{2}$

Question 3

Solve for b if $\frac{2}{3}b - 7 = b - 2(b + 3)$.

- (A) 0.6
- (B) 1.67
- (C) 3
- (D) 6
- (E) 7.8

Question 4

A company that produces tables finds that its monthly profit in dollars is given by the equation $P = 250x - 12,500$, where x is the number of tables that the company sells in a month. If the profit goal for April is \$10,000, how many tables must the company sell in order to reach the goal?

- (A) 60
- (B) 70
- (C) 80
- (D) 90
- (E) 100

Question 5

Solve for b if $3a + 5(a + b) = 7$.

- (A) $\frac{8a - 7}{5}$
- (B) $\frac{7 - 8a}{5}$
- (C) $\frac{7}{5} - 8a$
- (D) $8a - 7$
- (E) $7 - 8a$

Question 6

Allen is a furniture delivery driver whose daily income in dollars is given by the equation $I = 0.45m + 75$, where m is the number of miles that he drives in one day. If he made \$142.50 on Tuesday, how many miles did he drive that day?

- (A) 150
- (B) 175
- (C) 200
- (D) 225
- (E) 250

Question 7

What is the solution set of $3x - 5 = 13 - 6x$?

- (A) $\{-\frac{7}{3}\}$
- (B) $\{\frac{1}{2}\}$
- (C) $\{1\}$
- (D) $\{2\}$
- (E) $\{9\}$

Question 8

What is the solution set to the equation $\frac{4x - 3}{2} = 9$?

- (A) $\{10.5\}$
- (B) $\{5.25\}$
- (C) $\{1.875\}$
- (D) $\{1.5\}$
- (E) $\{6\}$

Question 9

What is the solution set to the equation $\frac{-6}{x+4} = 3$?

- (A) $\{-6\}$
- (B) $\{-18\}$
- (C) $\{-22\}$
- (D) $\{-4.5\}$
- (E) $\{5\}$

Question 10

If $m = \frac{n}{3} - 10$, then which of the following expresses n in terms of m ?

- (A) $n = 30m$
- (B) $n = \frac{10m}{3}$
- (C) $n = 3m + 10$
- (D) $n = \frac{m+10}{3}$
- (E) $n = 3m + 30$

**Hint for Question 1:**

Don't forget to distribute the minus sign.

Hint for Question 2:

Cross multiply first.

Hint for Question 3:

Don't forget to distribute the -2.

Treat the $\frac{2}{3}$ like you would any coefficient.

Hint for Question 4:

Substitute and solve for x.

Hint for Question 5:

Treat "a" like a constant.

Don't forget to distribute the 5.

Hint for Question 6:

You are given I and are looking for m.

You can test each answer choice or solve the equation.

Hint for Question 7:

Bring the x terms to one side and the constant terms to the other.

Hint for Question 8:

To solve for x, multiply both sides by 2 first.

You have the option of trying each answer choice.

Hint for Question 9:

To solve for x, cross-multiply.

Remember that $3 = \frac{3}{1}$.

You have the option of trying each answer choice.

Hint for Question 10:

Solve like you would any equation.

Treat m as a constant.



Answers!

(A) (B) (C) (D) (E)



Question 1.

The answer is (A).

Explanation for Question 1:

First simplify the left hand side of the equation.

Distribute the minus sign:

$$5 - x - 6 = 8$$

$$-x - 1 = 8$$

Add one to both sides of the equation:

$$-x - 1 + 1 = 8 + 1$$

$$-x = 9$$

Divide by -1:

$$x = -9$$

The answer is (A).

Question 2.

The answer is (E).

Explanation for Question 2:

Cross-multiply:

$$\frac{x+1}{5} = \frac{x}{3}$$

$$3(x+1) = 5x$$

Distribute the 3:

$$3x + 3 = 5x$$

Subtract 3x from both sides:

$$3 = 2x$$

Divide both sides by 2:

$$\frac{3}{2} = x$$

The answer is (E).

Question 3.

The answer is (A).

Explanation for Question 3:

One option is to fill in the answer choices;
however, this would have been more time consuming.

First distribute the -2:

$$\frac{2}{3}b - 7 = b - 2b - 6$$

Simplify the right hand side:

$$\frac{2}{3}b - 7 = -b - 6$$

Add b to both sides:

$$\frac{2}{3}b + b - 7 = -b + b - 6$$

$$\frac{2}{3}b + \frac{3}{3}b - 7 = -6$$

$$\frac{5}{3}b - 7 = -6$$

Add 7 to both sides:

$$\frac{5}{3}b = 1$$

Multiply both sides by $\frac{3}{5}$:

$$\frac{3}{5} \times \frac{5}{3}b = 1 \times \frac{3}{5}$$

$$b = \frac{3}{5} = 0.6$$

The answer is (A).

Question 4.

The answer is (D).

Explanation for Question 4:

$$250x - 12,500 = 10,000$$

$$250x = 22,500$$

$$x = 90$$

The answer is (D).

Question 5.

The answer is (B).

Explanation for Question 5:

First distribute the 5:

$$3a + 5a + 5b = 7$$

Combine the like terms:

$$8a + 5b = 7$$

Subtract $8a$ from both sides:

$$5b = 7 - 8a$$

Divide both sides by 5:

$$b = \frac{7-8a}{5}$$

The answer is (B).

Question 6.

The answer is (A).

Explanation for Question 6:

If you are going to try each answer choice, start with the middle answer.

$$(C) I = 0.45(200) + 75 = 165$$

This is too large so he must have driven fewer miles. Try (B).

$$(B) I = 0.45(175) + 75 = 153.75$$

This is still too large.

The answer is (A).

Question 7.

The answer is (D).

Explanation for Question 7:

Add $6x$ to both sides of the equation:

$$3x + 6x - 5 = 13 - 6x + 6x$$

$$9x - 5 = 13$$

Add 5 to both sides:

$$9x - 5 + 5 = 13 + 5$$

$$9x = 18$$

Divide both sides by 9:

$$x = 2$$

The answer is (D).

Question 8.

The answer is (B).

Explanation for Question 8:

To solve for x, first multiply both sides by 2:

$$2 \times \frac{4x-3}{2} = 9 \times 2$$

$$4x - 3 = 18$$

Add 3 to both sides:

$$4x = 21$$

Divide by 4:

$$x = 5.25$$

The answer is (B).

Question 9.

The answer is (A).

Explanation for Question 9:

To solve for x, cross multiply first:

$$\frac{-6}{x+4} = \frac{3}{1}$$

$$-6 = 3(x+4)$$

Don't forget to distribute the 3:

$$-6 = 3x + 12$$

Subtract 12 from both sides:

$$-18 = 3x$$

Divide by 3:

$$-6 = x$$

The answer is (A).

Question 10.

The answer is (E).

Explanation for Question 10:

To solve for n , first add 10 to both sides of the equation:

$$m + 10 = \frac{n}{3}$$

Now, multiply both sides by 3:

$$3 \times (m + 10) = \frac{n}{3} \times 3$$

Don't forget to distribute the 3:

$$3m + 30 = n$$

The answer is (E).



Question 1

If $\frac{x-3}{4} = \frac{1-x}{2}$, then $x = ?$

- (A) 5
- (B) $\frac{5}{3}$
- (C) $\frac{7}{3}$
- (D) $\frac{2}{3}$
- (E) This equation has no solution.

Question 2

If $10x + 4 = 6x - 4(x + 5)$, then $x = ?$

- (A) -1
- (B) -2
- (C) -3
- (D) $\frac{1}{8}$
- (E) $-\frac{1}{3}$

Question 3

If $x - (x - 1) - (x - 3) = 10$, then $x = ?$

- (A) -14
- (B) -6
- (C) -2
- (D) 0
- (E) 6

Question 4

If $\frac{y}{x+1} = \frac{3}{10}$, then which of the following equations gives y in terms of x ?

- (A) $y = \frac{3x}{10} + 1$
- (B) $y = \frac{10x + 10}{3}$
- (C) $y = 3x - 7$
- (D) $y = \frac{3x + 3}{10}$
- (E) $y = \frac{3}{10} - x - 1$

Question 5

Solve for b if $\frac{b}{2} + \frac{1}{2} = 1 - \frac{b}{4}$.

- (A) $-\frac{5}{7}$
- (B) $\frac{6}{7}$
- (C) 0
- (D) $\frac{1}{2}$
- (E) 7

Question 6

What is the solution set of the equation $\frac{1}{x} - \frac{3}{2} = \frac{7}{x}$?

- (A) $\{-4\}$
- (B) $\{-1\}$
- (C) $\{0\}$
- (D) $\{1\}$
- (E) $\{4\}$

Question 7

If $2m - 5 = 13$ and $6 - n = 8$, then $2m + n = ?$

- (A) 12
- (B) 14
- (C) 16
- (D) 18
- (E) 20

Question 8

To convert from degrees Celsius to degrees Fahrenheit the formula is

given by $F = \frac{9}{5}C + 32$. If the temperature is 85 degrees Fahrenheit,

what is the temperature in degrees Celsius?

- (A) 29.444...
- (B) 43
- (C) 70.777...
- (D) 95.4
- (E) 185

Question 9

If $\frac{2+x}{5+x} = \frac{2}{5} + \frac{1}{5}$, then $x = ?$

- (A) 1
- (B) 2.5
- (C) 5
- (D) -1.5
- (E) This equation has no solution.

Question 10

If $\frac{p}{2} - 9 = -7$ and $3q + 7 = 10$, then $p + q = ?$

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

**Hint for Question 1:**

Start by cross-multiplying.
Don't forget to distribute!

Hint for Question 2:

Distribute the -4 first.
You have the option of trying each answer choice.

Hint for Question 3:

Distribute the minus signs first.

Hint for Question 4:

Cross-multiply first. Treat x like a constant.

Hint for Question 5:

Find the lowest common denominator and multiply it through
to clear the equation of fractions.

Hint for Question 6:

Find the lowest common denominator and multiply it through
to clear the equation of fractions.
You have the option of trying each answer choice.

Hint for Question 7:

Find $2m$ and n and then add the two numbers.

Hint for Question 8:

Replace F with 85 and solve for C .
You have the option of trying each answer choice.

Hint for Question 9:

Combine the two fractions on the right hand side first.
Then cross-multiply.

Studyguide for the SAT Skill Quiz C: Rational Linear Equations

Hint for Question 10:

Solve each equation and then add the solutions.



Answers!



Question 1.

The answer is (B).

Explanation for Question 1:

Cross-multiply first:

$$2(x - 3) = 4(1 - x)$$

Distribute the 2 and 4:

$$2x - 6 = 4 - 4x$$

Add 4x to both sides:

$$6x - 6 = 4$$

Add 6 to both sides:

$$6x = 10$$

Divide by 6:

The answer is (B).

Question 2.

The answer is (C).

Explanation for Question 2:

Distribute the -4:

$$10x + 4 = 6x - 4x - 20$$

Combine the x terms on the right hand side:

$$10x + 4 = 2x - 20$$

Subtract 2x from both sides:

$$8x + 4 = -20$$

Subtract 4:

$$8x = -24$$

Divide by 8:

$$x = -3$$

The answer is (C).

Question 3.

The answer is (B).

Explanation for Question 3:

Distribute the minus signs first:

$$x - x + 1 - x + 3 = 10$$

Combine like terms:

$$-x + 4 = 10$$

Subtract 4 from both sides:

$$-x = 6$$

Divide by -1:

$$x = -6$$

The answer is (B).

Question 4.

The answer is (D).

Explanation for Question 4:

First cross-multiply:

$$10y = 3(x + 1)$$

$$10y = 3x + 3$$

Now divide by 10:

$$y = \frac{3x + 3}{10}$$

The answer is (D).

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Question 5.

The answer is (B).

Explanation for Question 5:

To get rid of the fractions multiply through by the LCD: 12.

Note that the 1 must be multiplied by the 12 too even though it is not a fraction.

$$12 \times \frac{b}{3} + 12 \times \frac{1}{2} = 12 \times 1 - 12 \times \frac{b}{4}$$

$$4b + 6 = 12 - 3b$$

Now add 3b to both sides:

$$7b + 6 = 12$$

Subtract 6 from both sides:

$$7b = 6$$

Divide by 7:

$$b = \frac{6}{7}$$

The answer is (B).

Question 6.

The answer is (A).

Explanation for Question 6:

To get rid of the fractions multiply through by the LCD: 2x.

$$2x \times \frac{1}{x} - 2x \times \frac{3}{2} = 2x \times \frac{7}{x}$$

$$2 - 3x = 14$$

Subtract 2 from both sides:

$$-3x = 12$$

Divide by -3:

$$x = -4$$

The answer is (A).

Studyguide for the SAT Skill Quiz C: Rational Linear Equations

Question 7.

The answer is (C).

Explanation for Question 7:

If $2m - 5 = 13$, then $2m = 18$. We don't need to know what m equals.

Now solve $6 - n = 8$ for n .

Subtract 6:

$$-n = 2$$

Divide by -1:

$$n = -2$$

$$2m + n = 18 + -2 = 16$$

The answer is (C).

Question 8.

The answer is (A).

Explanation for Question 8:

Replace F with 85:

$$85 = \frac{9}{5}C + 32$$

Subtract 32:

$$53 = \frac{9}{5}C$$

Multiply both sides by $\frac{5}{9}$:

$$\frac{5}{9} \times 53 = \frac{5}{9} \times \frac{9}{5}C$$

$$29.444... = C$$

The answer is (A).

Question 9.

The answer is (B).

Explanation for Question 9:

First add the two fractions:

$$\frac{2+x}{5+x} = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

$$\frac{2+x}{5+x} = \frac{3}{5}$$

Now, cross multiply:

$$5(2+x) = 3(5+x)$$

Distribute:

$$10 + 5x = 15 + 3x$$

Subtract $3x$ from both sides:

$$10 + 2x = 15$$

Subtract 10 from both sides:

$$2x = 5$$

Divide by 2:

$$x = 2.5$$

The answer is (B).

Studyguide for the SAT Skill Quiz C: Rational Linear Equations

Question 10.

The answer is (E).

Explanation for Question 10:

First find p.

Add 9 to both sides:

$$\frac{p}{2} = 2$$

Multiply both sides by 2:

$$2 \times \frac{p}{2} = 2 \times 2$$

$$p = 4$$

Now find q.

Subtract 7 from both sides:

$$3q = 3$$

Divide by 3:

$$q = 1$$

$$p + q = 4 + 1 = 5$$

The answer is (E).



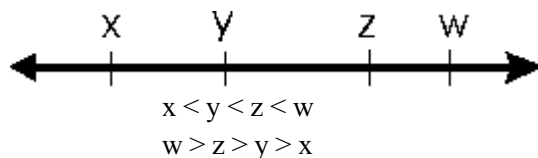
Review Time

INEQUALITIES

Inequalities aren't so bad if you use a number line.

Numbers to the left are always $<$ numbers to the right.

Numbers to the right are always $>$ numbers to the left.



Some Important Relationships

1. If $x < y$, then $x + a < y + a$ for all values of a .

This means when you add a number to both sides of an inequality the order remains the same.

For example,

$$\begin{aligned}x - 5 &< 7 \\x - 5 + 5 &< 7 + 5 \\x &< 12\end{aligned}$$

2. If $x < y$ and a is positive, then $xa < ya$.

This means when you multiply both sides of an inequality by a positive number the order remains the same.

For example,

$$\begin{aligned}2x &> 10 \\2x(1/2) &> 10(1/2) \\x &> 5\end{aligned}$$

Studyguide for the SAT Skill Review: Inequalities

3. If $x < y$ and a is negative, then $xa > ya$.

This means when you multiply both sides of an inequality by a negative number the order changes.

For example,

$$-3x < 9$$

$$-3x(-1/3) < 9(-1/3)$$

$$x > -3$$

4. For a positive number x , if $x > 1$, then $x^2 > x$.

For example,

$$x = 3, \text{ then } 3^2 > 3, \text{ so } 9 > 3$$

5. For a positive number x , if $x < 1$, then $x^2 < x$.

For example,

$$x = 1/2, \text{ then } (1/2)^2 < 1/2, \text{ so } 1/4 < 1/2$$

Some inequalities are combined inequalities. $5 < x$ and $x < 7$ can be combined as $5 < x < 7$.

Note: For combined inequalities, the order for both original inequalities is the same and the x is in the "middle."

For example: $x > 3$ and $7 < x$ cannot be combined.



Question 1

If $1 < x < 5$ and $3 < y < 10$, then

- (A) $1 < x + y < 10$
- (B) $3 < x + y < 5$
- (C) $4 < x + y < 15$
- (D) $5 < x + y < 10$
- (E) $10 < x + y$

Question 2

If $33 < x < 43$, then how many distinct values could x have?

- (A) 9
- (B) 10
- (C) 11
- (D) 12
- (E) None of the above

Question 3

If $-12 > -x$ and $y > 7$, then which answer below is a possible value for $(x + y)$?

- (A) -13
- (B) -5
- (C) 19
- (D) 20
- (E) All of the above

Question 4

If $a > b$, which of the following must be true?

- (A) $a^2 > b^2$
- (B) $-a > -b$
- (C) $ab > b^2$
- (D) $a > 3b$
- (E) $-a < -b$

Question 5

If $2x + 3 > 3x - 10$, then which of the following is a possible value for $x + 2$?

- (A) 13
- (B) 15
- (C) 16
- (D) 18
- (E) 20

Question 6

If $1 < x < 8$, which of the following is a possible value for x ?

- (A) 1
- (B) 5
- (C) 9
- (D) 11
- (E) It cannot be determined from the given information.

Question 7

If $3x < 18 < 4x$, then which of the following is NOT a possible value for x ?

- (A) 4.3
- (B) 4.6
- (C) 5.1
- (D) 5.6
- (E) 5.9

Question 8

If $2 \leq n < m \leq 8$ and n and m are integers, what is the least possible value of $(n + m)/mn$?

- (A) $5/56$
- (B) $16/64$
- (C) $15/56$
- (D) $7/10$
- (E) $5/6$

Question 9

Boxes A, B, and C contain marbles. Box A contains more marbles than box B, and box B contains more marbles than Box C. If box B contains 10 marbles, which of the following could not be the total number of marbles in boxes A and B?

- (A) 20
- (B) 21
- (C) 22
- (D) 23
- (E) 24

Question 10

Which of the following is equivalent to $\frac{|x - 3|}{|2|} > 15$?

- (A) $x > 33$
- (B) $x > 36$
- (C) $x < -27$ or $x > 33$
- (D) $x < -12$ or $x > 36$
- (E) $-4.5 < x < 10.5$

Studyguide for the SAT Skill Quiz A: Inequalities



Hint for Question 1:

Add the two inequalities together.

Hint for Question 2:

Are there any restrictions placed on the kinds of numbers for x ?

Hint for Question 3:

Get x without the negative and on the left side of the first inequality before trying to answer this question.

Hint for Question 4:

You have to think hard about possible replacement numbers for a and b .

Hint for Question 5:

Solve the inequality for x , then add 2 to find the inequality for $x + 2$. Use this inequality to answer the question.

Hint for Question 6:

This inequality means x is between 1 and 8.

Hint for Question 7:

Use backsolving. Substitute in each answer choice for x until you find the number that answers the question.

Hint for Question 8:

Use backsolving or write the list of all possible solutions for $(n + m)/mn$.

Hint for Question 9:

What is the least number of marbles that could be in box A? The total for this number and the number of marbles in box B is the least number for the sum.

Studyguide for the SAT Skill Quiz A: Inequalities

Hint for Question 10:

Try numbers for x .

Remember that trying one number may eliminate multiple answer choices.



Question 1.

The answer is (C).

Explanation for Question 1:

Line up the inequalities to make the addition easier.

$$1 < x < 5$$

$3 < y < 10$ The order signs must be the same in both inequalities to do this.

$4 < x + y < 15$. The answer is (C).

Question 2.

The answer is (E).

Explanation for Question 2:

The question does not state that x is an integer. There are infinite solutions for x .

The answer is (E).

Question 3.

The answer is (D).

Explanation for Question 3:

Rewrite $-12 > -x$ as $-x < -12$. To solve for x you must multiply by -1 . When you multiply by a negative the order changes so $x > 12$. Now with $x > 12$ and $y > 7$, $x + y > 19$.

The answer is (D).

Question 4.

The answer is (E).

Explanation for Question 4:

Let's look at each choice.

(A) If you choose $a = -1$ and $b = -2$, $a > b$ but $a^2 < b^2$. ($1 < 4$) Not the answer.

(B) If you choose $a = 2$ and $b = 1$, $a > b$ but $-a < -b$. ($-2 < -1$) Not the answer.

(C) If you choose $a = 2$ and $b = -2$, $a > b$ but $ab < b^2$. ($-4 < 4$) Not the answer.

(D) If you choose $a = 2$ and $b = 1$, $a > b$ but $a < 3b$. ($2 < 3$) Not the answer.

(E) No matter what numbers you choose for a and b , $a > b$, $-a$ is always less than $-b$.

The answer is (E).

Remember, when you multiply an inequality by a negative the order changes.

Question 5.

The answer is (A).

Explanation for Question 5:

Solving the inequality for x : $13 > x$ or $x < 13$; this means $x + 2 < 15$. The only answer choice less than 15 is 13. The answer is (A).

Studyguide for the SAT Skill Quiz A: Inequalities

Question 6.

The answer is (B).

Explanation for Question 6:

The only answer choice between 1 and 8 is 5. The answer is (B).

Question 7.

The answer is (A).

Explanation for Question 7:

Substitute in answer choice (A) $12.9 < 18 < 17.2$ Oops. This doesn't work.

The answer is (A). Just for a check look at the following.

Choice (B) $13.8 < 18 < 18.4$

Choice (C) $15.3 < 18 < 20.4$

Choice (D) $16.8 < 18 < 22.4$

Choice (E) $17.7 < 18 < 23.6$

On the test have confidence in your ability enough so that once you find the answer you do not need to check the rest. You want to move to the next problem as quickly as possible.

Studyguide for the SAT Skill Quiz A: Inequalities

Question 8.

The answer is (C).

Explanation for Question 8:

The quick approach is to look at each answer choice, see if it is a possible solution for $(n + m)/mn$, and if it is select the smallest answer choice.

- (A) Can $5/56$ be a solution? No
- (B) Can $16/64$ be a solution? No
- (C) Can $15/56$ be a solution? Yes $n = 7$, $m = 8$.
- (D) Can $7/10$ be a solution? Yes $n = 2$, $m = 5$.
- (E) Can $5/6$ be a solution? Yes $n = 2$, $m = 3$.

Out of our solutions $7/10$, $15/56$, and $5/6$, the smallest number is $15/56$. You can change each to a decimal to determine the smallest number. The answer is (C). Another, perhaps, longer way is to solve the problem as follows.

To write the list and make the following table:

n	m	$(n + m)/mn$	
2	3	$5/6$	
	4	$6/8 = 3/4$	
	5	$7/10$	
	6	$8/12 = 2/3$	
	7	$9/14$	There is a pattern here. Notice as this list continues each succeeding fraction is smaller. The smallest fraction should be the last one if this list continues.
3	8	$10/16 = 5/8$	
	4	$7/12$	
	5	$8/15$	
	6	$9/18 = 1/2$	
	7	$10/21$	
4	8	$11/24$	
	5	$9/20$	
	6	$10/24 = 5/12$	
	7	$11/28$	
	8	$12/32 = 3/8$	

For the last fraction $n = 7$ and $m = 8$. The fraction will be $15/56$. The answer is (C).

Question 9.

The answer is (A).

Explanation for Question 9:

The least number of marbles in box A would be 11. The least sum of marbles in box A and B is $10 + 11 = 21$.

All numbers greater than 21 are also possible sums.

The answer is (A).

Question 10.

The answer is (C).

Explanation for Question 10:

The absolute value of 2 is 2.

$$|x - 3| > 30$$

$$x > 33 \text{ or } x < -27.$$

The answer is (C).



Question 1

If $\frac{3}{x} \leq \frac{2}{3}$, then what is the smallest possible value of x ?

- (A) 1.5
- (B) 2
- (C) 4.5
- (D) 5
- (E) 9

Question 2

If $a < b < 0$, then which of the following *must* be true?

- (A) $ab < 0$
- (B) $a^2 < b^2$
- (C) $b - a < 0$
- (D) $a + b < 0$
- (E) $a - b < a$

Question 3

If $0 < x < 1$, then all of the following *must* be true EXCEPT

- (A) $0 < x^2 < 1$
- (B) $\frac{1}{x} > 1$
- (C) $-1 < -x < 0$
- (D) $x^3 > 1$
- (E) $1 < x + 1 < 2$

Question 4

If $\frac{1}{x} > \frac{2}{x+3}$, then which of the following

is the largest possible value of x ?

- (A) -3
- (B) -2
- (C) 2
- (D) 3
- (E) 5

Question 5

If $|2x + 5| \leq 9$, then which of the following is the smallest negative number in the solution set?

- (A) -9
- (B) -7
- (C) -4.5
- (D) -2
- (E) -0.5

Question 6

Which of the following is the solution to $|\frac{x}{2} + 8| < 11$?

- (A) $-38 < x < 6$
- (B) $-30 < x < 14$
- (C) $0 < x < 6$
- (D) $x < 6$
- (E) $x < 14$

Question 7

Which of the following is the solution set of $x^2 < 9$?

- (A) $x < 3$
- (B) $x > 3$
- (C) $x < -3$
- (D) $-3 < x < 3$
- (E) $x < -3$ or $x > 3$

Question 8

Which of the following is not part of the solution set of $|5 - x| \geq 12$?

- (A) -17
- (B) -7
- (C) -5
- (D) 17
- (E) 25

Question 9

Which of the following is not part of the solution set of $|3x - 1| < 9$?

- (A) -3
- (B) -1
- (C) 0
- (D) 1
- (E) 3

Question 10

Which is the solution set of $-4(x + 3) + x < x - 6$?

- (A) $x > -1.5$
- (B) $x < -1.5$
- (C) $x > -3$
- (D) $x > 2.25$
- (E) $x < 2.25$

**Hint for Question 1:**

Multiply both sides by the LCD.

Hint for Question 2:

Pick numbers for a and b .
You may have to pick a number more than once in order to find the answer.

Hint for Question 3:

Pick a number for x .

Hint for Question 4:

Multiply both sides by the LCD.

Hint for Question 5:

Try each answer choice until you get one to work.
Remember that you are looking for the smallest number that will work.

Hint for Question 6:

Try numbers for x .
Remember that trying one number may eliminate multiple answer choices.

Hint for Question 7:

Try numbers for x .
Remember that trying one number may eliminate multiple answer choices.

Hint for Question 8:

Try each answer choice.
Remember that the absolute value of a negative number is a positive.

Hint for Question 9:

Try each answer choice.
Remember that the absolute value of a negative number is a positive.

Studyguide for the SAT Skill Quiz B: Inequalities

Hint for Question 10:

Remember to switch the direction of the inequality if you multiply or divide by a negative number.



Question 1.

The answer is (C).

Explanation for Question 1:

To clear the inequality of fractions, multiply both sides by the LCD: $3x$.

$$3x \times \frac{3}{x} \leq \frac{2}{3} \times 3x$$

$$9 \leq 2x$$

Divide both sides by 2:

$$4.5 \leq x \text{ or } x \geq 4.5$$

This means that x must be 4.5 or greater so 4.5 is the smallest possible value of x .

The answer is (C).

Question 2.

The answer is (D).

Explanation for Question 2:

a and b must be negative because $a < b < 0$.

Let $a = -5$ and $b = -1$:

(A) $(-5)(-1) < 0$

$5 < 0$ This is false.

(B) $(-5)^2 < (-1)^2$

$25 < 1$ This is false.

(C) $-1 - (-5) < 0$

$-1 + 5 < 0$

$4 < 0$ This is false.

(D) $-5 + -1 < 0$

$-6 < 0$ This is true.

(E) $-5 - (-1) < -5$

$-5 + 1 < -5$

$-4 < -5$ This is false.

The answer is (D).

Studyguide for the SAT Skill Quiz B: Inequalities

Question 3.

The answer is (D).

Explanation for Question 3:

$$\text{Let } x = \frac{1}{2}$$

(A) $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, which is between 0 and 1. This is true.

(B) $\frac{1}{1/2} = 2$, which is > 1 . This is true.

(C) $-\frac{1}{2}$ is between -1 and 0. This is true.

(D) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, which is not greater than 1. This is false.

(E) $\frac{1}{2} + 1 = \frac{3}{2}$, which is between 1 and 2. This is true.

The answer is (D).

Question 4.

The answer is (D).

Explanation for Question 4:

To clear the inequality of fractions,
multiply both sides by the LCD: $x(x + 3)$.

$$x(x + 3) \times \frac{1}{x} \geq \frac{2}{x+3} \times x(x + 3)$$

$$x + 3 \geq 2x$$

Subtract x from both sides:

$$3 \geq x$$

Because x must be less than or equal to 3,
3 is the largest possible value of x .

The answer is (D).

Question 5.

The answer is (B).

Explanation for Question 5:

Try each answer choice.

$$(A) |2 \times (-9) + 5| = |-18 + 5| = |-13| = 13$$

13 is not ≤ 9

$$(B) |2 \times (-7) + 5| = |-14 + 5| = |-9| = 9$$

$$9 \leq 9$$

The answer is (B).

Note that choices (C), (D), and (E) are all part of the solution set.

However, none of these is the smallest negative number in the solution set.

Question 6.

The answer is (A).

Explanation for Question 6:

Try $x = 7$:

$$\left|\frac{7}{2} + 8\right| = |11.5| = 11.5,$$

which is not < 11

Because 7 is not part of the solution set,

Choices (B) and (E) cannot be the answer.

Try $x = -40$:

$$\left|\frac{-40}{2} + 8\right| = |-20 + 8| = |-12| = 12,$$

which is not < 11

Because -40 is not part of the solution set,

Choice (D) cannot be the answer.

Try $x = -2$:

$$\left|\frac{-2}{2} + 8\right| = |-1 + 8| = 7$$
$$7 < 11$$

Because -2 is part of the solution set,

Choice (C) cannot be the answer.

The answer is (A).

Question 7.

The answer is (D).

Explanation for Question 7:

Try $x = 0$:

$$0^2 = 0, \text{ which is } < 9$$

Because 0 is part of the solution set,
choices (B), (C), and (E) cannot be the answer.

Try $x = -4$:

$$(-4)^2 = 16, \text{ which is not } < 9$$

Because -4 is not part of the solution set,
choice (A) cannot be the answer.

The answer is (D).

Question 8.

The answer is (C).

Explanation for Question 8:

Try each answer:

$$(A) |5 - (-17)| = |5 + 17| = 22, \text{ which is } \geq 12$$

$$(B) |5 - (-7)| = |5 + 7| = 12, \text{ which is } \geq 12$$

$$(C) |5 - (-5)| = |5 + 5| = 10, \text{ which is not } \geq 12$$

The answer is (C).

Question 9.

The answer is (E).

Explanation for Question 9:

Try each answer:

$$(A) |3 \times (-3) + 1| = |-8| = 8, \text{ which is } < 9$$

$$(B) |3 \times (-1) + 1| = |-2| = 2, \text{ which is } < 9$$

$$(C) |3 \times (0) + 1| = |1| = 1, \text{ which is } < 9$$

$$(D) |3 \times (1) + 1| = |4| = 4, \text{ which is } < 9$$

The answer is (E). There is no need to try it.

Question 10.

The answer is (A).

Explanation for Question 10:

Solve this as you would an equation:

$$-4x - 12 + x < x - 6$$

$$-3x - 12 < x - 6$$

$$-4x - 12 < -6$$

$$-4x < 6$$

$$x > -1.5$$

Note: The direction of the inequality switches when you divide by -4.

The answer is (A).



FACTOR THE EXPRESSION

You need to be able to factor common algebraic expressions like these:

$$x^2 + ax = x(x + a)$$

$$x^2 - a^2 = (x + a)(x - a)$$

$$x^2 + 2ax + a^2 = (x + a)^2$$

FOIL

Use FOIL to factor. FOIL stands for **F**irst, **O**uter, **I**nnner, **L**ast.

You use FOIL when multiplying two binomials.

$$(a + b)(c + d)$$

The **F**irst represents the first variable in each binomial, a and c.

The **O**uter represents the two outside variables, a and d.

The **I**nnner represents the two inside variables, b and c.

The **L**ast represents the last variables in each binomial, b and d.

Using FOIL $(a + b)(c + d) = ac + ad + bc + bd$.

For example, $(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$.

You have to add like terms.

An expression with the leading term having a power of 2 is called a quadratic expression. When you factor a quadratic expression you express it as a product of two expressions. To check your factoring, you sometimes have to use FOIL.

For example, factor $x^2 - 5x + 6 = (x - 2)(x - 3)$. Multiply using FOIL to see if the factoring is correct.

Studyguide for the SAT Skill Review: Factor the Expression

Example 1:

If $x^2 + xy = 3(x + y)$, what is the value of x ?

Solution:

$$x^2 + xy = 3(x + y)$$

$$x(x + y) = 3(x + y) \text{ Divide both sides by } x + y$$

$$\frac{x(x + y)}{x + y} = \frac{3(x + y)}{x + y}$$

$$x = 3$$

Example 2:

If $x > 0$ and $(x + 1)(x - 1) = 8$, what is the value of x ?

Solution:

$$(x + 1)(x - 1) = 8$$

$$x^2 - 1 = 8$$

$$x^2 = 9$$

$x = \pm 3$ The answer is 3 because x must be positive.

Example 3:

If $x^2 + kx + 25 = (x + n)^2$, what are the values of k and n ?

Solution:

This type of problem is solved by a process that is called completing the square. Look at the following examples.

$$x^2 + 8x + 16 = (x + ?)^2 \text{ To make } x + ? \text{ a perfect square the } ? = 4.$$

$$x^2 + 10x + 25 = (x + ?)^2 \text{ To make } x + ? \text{ a perfect square the } ? = 5.$$

You find the $?$ by taking the square root of the last term.

Now, how do you find the coefficient of the middle term?

$$x^2 + *x + 49 = (x + ?)^2 \text{ To find the } * \text{ you first have to find the } ?$$

The $? = 7$ and $* = 14$.

$$\text{Remember } (x + a)^2 = x^2 + 2ax + a^2$$

Studyguide for the SAT Skill Review: Factor the Expression

$x^2 + *x + 81 = (x + ?)^2$. To find the * you have to find the ?
The ? = 9 and * = 18.

Remember the relationships.

Back to our example: If $x^2 + kx + 25 = (x + n)^2$, what are the values of k and n? n must be equal the square root of 25 so n = 5. If n = 5, then k = 10.

One more thing needs to be mentioned. Often you will be asked to find the value of an expression. Try to find the value of the expression directly and not spend time finding the value of each term in the expression.

Example 4:

If $x^2 + y^2 = 13$ and $xy = 6$, what is the value of $x + y$?

Solution:

You could spend time solving for x and then for y and then finally adding x and y to get the answer. The equations to solve will be difficult. Begin by looking for an easier way.

$(x + y)^2 = x^2 + 2xy + y^2$. This equation has $x^2 + y^2$ and xy in it.

Substitute in the values for each of these expressions. $x^2 + y^2 = 13$ and $xy = 6$

so, $(x + y)^2 = x^2 + y^2 + 2xy = 13 + 2(6) = 13 + 12 = 25$

$(x + y)^2 = 25$ and $x + y = \pm 5$.



Question 1

If $x^2 - y^2 = 35$ and $x + y = 5$, what is the value of $x - y$?

- (A) 5
- (B) 7
- (C) 30
- (D) 35
- (E) 40

Question 2

If $x + 2y = 8$ and $2x - y = 2$, what is the value of $2x^2 + 3xy - 2y^2$?

- (A) 2
- (B) 4
- (C) 6
- (D) 10
- (E) 16

Question 3

If the length of a rectangle is $x - 5$ and the width is $x - 2$, which of the following represents the area of the rectangle?

- (A) $x^2 - 7x + 10$
- (B) $x^2 + 7x - 10$
- (C) $x^2 - 3x + 10$
- (D) $x^2 - 3x - 10$
- (E) $x^2 - 7x - 10$

Question 4

If $x^2 + y^2 = 18$ and $xy = -9$, what is the positive value of $x - y$?

- (A) 3
- (B) $3\sqrt{3}$
- (C) 6
- (D) 27
- (E) 36

Question 5

If $x^2 + 3x + 4 = 32$ and $x < 0$, what is the value of x ?

- (A) -4
- (B) -7
- (C) -8
- (D) -28
- (E) -35

Question 6

What is the value of k , if $25x^2 + 2kx + 16$ is a perfect square?

- (A) 4
- (B) 16
- (C) 20
- (D) 40
- (E) 41

Question 7

If a rectangle has an area of $2x^2 + x - 3$ and a width of $2x + 3$, which of the following represents the length of the rectangle?

- (A) $2x - 1$
- (B) $x - 1$
- (C) $x + 1$
- (D) $x - 2$
- (E) $x + 2$

Question 8

Simplify $(x + 3)(2x - 5) + (x - 1)(x - 3)$.

- (A) $3(x^2 + x - 4)$
- (B) $3x^2 + 19x + 18$
- (C) $3(x^2 - 2x - 4)$
- (D) $3(x^2 - x - 4)$
- (E) $3x^2 - 3x + 18$

Question 9

If $x^2 + kx + 64 = (x + r)^2$ and $r > 0$, what is the value of k ?

- (A) 64
- (B) 32
- (C) 16
- (D) 8
- (E) 4

Question 10

If $49x^2 + kx + 16 = (7x + n)^2$ and $n > 0$, what is the value of $k + n$?

- (A) 60
- (B) 56
- (C) 32
- (D) 28
- (E) 4

Studyguide for the SAT Skill Quiz A: Factor the Expression



Hint for Question 1:

Factor $x^2 - y^2$ and divide.

Hint for Question 2:

What can you do with the two binomials to get the quadratic expression?

Hint for Question 3:

What is the formula for the area of a rectangle?

Hint for Question 4:

Square $x - y$ to find this solution.

Hint for Question 5:

Set the equation equal to 0 and factor to solve for x .

Hint for Question 6:

This is another complete the square problem. Be careful!

Hint for Question 7:

If you know the area and width of a rectangle, what operation do you use to find the length?

Hint for Question 8:

Multiply and simplify.

Hint for Question 9:

This is another complete the square problem.

Hint for Question 10:

This is another complete the square problem.



Answers!

(A) (B) (C) (D) (E)



Question 1.

The answer is (B).

Explanation for Question 1:

$$x^2 - y^2 = (x + y)(x - y)$$

$$(x + y)(x - y) = 35$$

$$\frac{(x + y)(x - y)}{x + y} = \frac{35}{5}$$

$$x - y = 7$$

The answer is (B).

Question 2.

The answer is (E).

Explanation for Question 2:

$$2x^2 + 3xy - 2y^2 = (x + 2y)(2x - y)$$

$$(x + 2y)(2x - y) = 8(2)$$

$$2x^2 + 3xy - 2y^2 = 16$$

The answer is (E).

Question 3.

The answer is (A).

Explanation for Question 3:

The area is $(x - 5)(x - 2) = x^2 - 7x + 10$. The answer is (A).

Question 4.

The answer is (C).

Explanation for Question 4:

$$(x - y)^2 = x^2 - 2xy + y^2 = 18 - (-18) = 36$$

$$(x - y)^2 = 36$$

$$x - y = \pm 6$$

The answer is (C).

Question 5.

The answer is (B).

Explanation for Question 5:

$$x^2 + 3x + 4 = 32$$

$$x^2 + 3x - 28 = 0$$

$$(x + 7)(x - 4) = 0$$

$$x = -7, \text{ or } x = 4.$$

Because $x < 0$, $x = -7$.

The answer is (B). For this problem you could backsolve by replacing x in the original equation with each of the answer choices until you found the one that solved the equation.

Question 6.

The answer is (C).

Explanation for Question 6:

$$25x^2 + 2kx + 16 = (5x + 4)^2$$

$$(5x + 4)^2 = 25x^2 + 40x + 16 \text{ so } 2k = 40 \text{ and } k = 20.$$

The answer is (C).

Question 7.

The answer is (B).

Explanation for Question 7:

Divide $2x^2 + x - 3$ by $2x + 3$.

$$\frac{2x^2 + x - 3}{2x + 3} = \frac{(2x + 3)(x - 1)}{2x + 3} = x - 1$$

The answer is (B).

Question 8.

The answer is (D).

Explanation for Question 8:

$$(x + 3)(2x - 5) + (x - 1)(x - 3) = 2x^2 + x - 15 + x^2 - 4x + 3 =$$

$$3x^2 - 3x - 12 = 3(x^2 - x - 4)$$

The answer is (D).

Question 9.

The answer is (C).

Explanation for Question 9:

$$x^2 + kx + 64 = (x + 8)^2$$

$$(x + 8)^2 = x^2 + 16x + 64. \text{ The value for } k \text{ is } 16. \text{ The answer is (C).}$$

Question 10.

The answer is (A).

Explanation for Question 10:

Factoring $49x^2 + kx + 16$ gives you $(7x + 4)^2$.

Therefore, $(7x + 4)^2 = (7x + n)^2$, so $n = 4$.

$(7x + 4)^2 = 49x^2 + kx + 16 = 49x^2 + 56x + 16$, so
 $k = 56$. $k + n = 56 + 4 = 60$. The answer is (A).



Question 1

If $4x^2 + kx + 49 = (2x + r)^2$ and $k > 0$, what is the value of r ?

- (A) 49
- (B) 28
- (C) 14
- (D) 7
- (E) $7/2$

Question 2

If $x^2 - y^2 = 8$ and $x + y = 2$, what is the value of $x - y$?

- (A) 16
- (B) 8
- (C) 6
- (D) 4
- (E) 2

Question 3

If $x - y = 3$ and $x^2 - y^2 = 21$, what is the value of $x + y$?

- (A) $1/7$
- (B) 7
- (C) 15
- (D) 18
- (E) 24

Question 4

What is the value of k , if $4x^2 + kx + 49$ is a perfect square?

- (A) 56
- (B) 28
- (C) 14
- (D) 7
- (E) 2

Question 5

$(103)(97) =$

- (A) $100^2 - 2^2$
- (B) $100^2 + 5^2$
- (C) $101^2 - 2^2$
- (D) $100^2 - 3^2$
- (E) $102^2 - 98^2$

Question 6

If a rectangle has an area of $x^2 + 5x + 6$ and length of $x + 2$, which of the following represents the width of the rectangle?

- (A) x
- (B) $x + 1$
- (C) $x + 2$
- (D) $x + 3$
- (E) $x + 4$

Question 7

Simplify $(x - 4)(x + 1) - (x^2 - 3x)$.

- (A) $2x^2 - 6x - 4$
- (B) $-7x - 4$
- (C) $-6x - 4$
- (D) -4
- (E) 4

Question 8

If $(x + 2)(x + 4) - (x - 6)(x + 2) = 0$, what is the value of x ?

- (A) -2
- (B) -4
- (C) 0
- (D) 4
- (E) 6

Question 9

A rectangle with area $x^2 - 1$ has one side with a length $x + 1$. What is the length of an adjacent side?

- (A) $x - 1$
- (B) x
- (C) $x + 1$
- (D) $x^2 - x - 2$
- (E) $x^2 + x$

Question 10

If $16x^2 + kx + 25 = (4x + n)^2$ and $k > 0$, what is the value of $k - n$?

- (A) 5
- (B) 16
- (C) 25
- (D) 35
- (E) 40

Studyguide for the SAT Skill Quiz B: Factor the Expression



Hint for Question 1:

This is another completing the square problem.
Finding r is easier than finding k .

Hint for Question 2:

You can factor $x^2 - y^2$ and then divide.

Hint for Question 3:

You can factor $x^2 - y^2$ and then divide.

Hint for Question 4:

This is another way to ask you to complete the square and then find k .

Hint for Question 5:

Think of a way to write $(103)(97)$ in the form $(x + y)(x - y)$.

Hint for Question 6:

If you know the area of a rectangle and the length, what operation do you use to find the width? Factor and then divide.

Hint for Question 7:

Multiply and then subtract. Don't forget to distribute the negative.

Hint for Question 8:

Multiply and then subtract. Don't forget to distribute the negative.

Hint for Question 9:

If you know the area and one side of a rectangle, what operation do you use to find the other side? Factor and then divide.

Hint for Question 10:

This is another completing the square problem. Just remember you need the numbers for both k and n .



(A) (B) (C) (D) (E)

Question 1.

The answer is (D).

Explanation for Question 1:

To complete the square, notice that r is the square root of 49. $r = 7$.

The answer is (D).

Question 2.

The answer is (D).

Explanation for Question 2:

$x^2 - y^2 = (x + y)(x - y)$. Use this fact to solve for $x - y$.
 $(x + y)(x - y) = 8$

$\frac{(x + y)(x - y)}{x + y} = \frac{8}{2}$. Divide the equation by the equation $x + y = 2$.

$$x - y = 4$$

The answer is (D).

Question 3.

The answer is (B).

Explanation for Question 3:

$x^2 - y^2 = (x + y)(x - y)$. Use this fact to solve for $x + y$.
 $(x + y)(x - y) = 21$

$\frac{(x + y)(x - y)}{x - y} = \frac{21}{3}$. Divide the equation by the equation $x - y = 3$.

$$x + y = 7$$

The answer is (B).

Question 4.

The answer is (B).

Explanation for Question 4:

To find the value of k , first find the perfect square for $4x^2 + kx + 49$.

$$4x^2 + kx + 49 = (2x + ?)^2 \quad ? = 7 \quad (\text{Don't forget the } 2x!)$$

$$4x^2 + kx + 49 = (2x + 7)^2. \text{ From } (2x + 7)^2 = 4x^2 + 28x + 49$$
$$k = 28$$

The answer is (B).

Studyguide for the SAT Skill Quiz B: Factor the Expression

Question 5.

The answer is (D).

Explanation for Question 5:

Use the hint to rewrite 103 as $100 + 3$ and rewrite 97 as $100 - 3$.

$$(103)(97) = (100 + 3)(100 - 3) = 100^2 - 3^2$$

The answer is (D).

Question 6.

The answer is (D).

Explanation for Question 6:

$$\text{The width would be } \frac{x^2 + 5x + 6}{x + 2} = \frac{(x + 3)(x + 2)}{x + 2}$$

The answer is (D).

Question 7.

The answer is (D).

Explanation for Question 7:

$$(x - 4)(x + 1) - (x^2 - 3x) = x^2 - 3x - 4 - x^2 + 3x = -4.$$

The answer is (D).

Question 8.

The answer is (A).

Explanation for Question 8:

There are numerous ways to solve this equation.

Here's one way.

$$(x + 2)(x + 4) - (x - 6)(x + 2) = 0. \text{ Factor out an } x + 2.$$

$$(x + 2)(x + 4 - (x - 6)) = 0$$

$$(x + 2)(x + 4 - x + 6) = 0$$

$$(x + 2)(10) = 0$$

$$x = -2$$

Here's another solution.

$$(x + 2)(x + 4) - (x - 6)(x + 2) = 0. \text{ Multiply and subtract.}$$

$$x^2 + 6x + 8 - (x^2 - 4x - 12) = 0$$

$$x^2 + 6x + 8 - x^2 + 4x + 12 = 0$$

$$10x + 20 = 0$$

$$10x = -20$$

$$x = -2$$

Whatever you do, don't divide the original equation by $x + 2$.

In this problem you'd be dividing by 0. The answer is (A).

You also could use backsolving.

Question 9.

The answer is (A).

Explanation for Question 9:

Divide $x^2 - 1$ by $x + 1$.

$$\frac{x^2 - 1}{x + 1} = \frac{(x + 1)(x - 1)}{x + 1} = x - 1$$

The answer is (A).

Question 10.

The answer is (D).

Explanation for Question 10:

$$16x^2 + kx + 25 = (4x + n)^2. \text{ By completing the square you should find that } n = 5.$$

$$\text{Then } k \text{ must equal } 40! \quad 16x^2 + 40x + 25 = (4x + 5)^2$$

The value for $k - n$ is $40 - 5 = 35$. The answer is (D).



PROPORTIONS

A proportion compares two ratios. For example $a:b$ as $c:d$.

You read this proportion as "a is to b as c is to d."

You can write this proportion as the equation $\frac{a}{b} = \frac{c}{d}$.

If one term of the proportion is missing you can find it by cross-multiplying.

Example 1:

Solve for x. $6:x$ as $9:12$

Write the proportion as an equation.

Solution:

$$\frac{6}{x} = \frac{9}{12}$$

$$72 = 9x$$

$$8 = x$$

Proportions can be used to solve many word problems. Here is an example.

Example 2:

A cookie recipe calls for $\frac{3}{4}$ cup of sugar to make 36 three-inch cookies.

According to this recipe, how many cups of sugar should be used to make 80 three-inch cookies?

- (A) $\frac{3}{5}$ (B) $1\frac{1}{2}$ (C) $1\frac{2}{3}$ (D) $2\frac{1}{2}$ (E) 3

Studyguide for the SAT Skill Review: Proportions

Solution:

To solve this problem, set up a proportion. $\frac{3}{4}$ is to 36 as x is to 80.

$$\frac{\frac{3}{4}}{36} = \frac{x}{80}$$

$$80\left(\frac{3}{4}\right) = 36x$$

$$60 = 36x$$

$$\frac{60}{36} = x$$

$$1\frac{2}{3} = x \quad \text{The answer is (C).}$$



Question 1

Solve for x. $\frac{5}{6} = \frac{15}{x}$

- (A) 3
- (B) 5
- (C) 18
- (D) 30
- (E) 90

Question 2

Solve for the missing number. $\frac{6}{9} = \frac{x}{12}$

- (A) 2
- (B) 8
- (C) 54
- (D) 72
- (E) 108

Question 3

Alesia wants to enlarge her 4-inch-wide by 6-inch-long photograph.

She wants the new width to be 10 inches.

To keep the same proportion, what must be the length of the new photograph?

- (A) 10
- (B) 12
- (C) 15
- (D) 25
- (E) 60

Studyguide for the SAT Skill Quiz A: Proportions

Question 4

On a certain map 2 centimeters equal 5 miles.
How many centimeters are equal to 34 miles?

- (A) 5
- (B) 12.8
- (C) 13
- (D) 13.6
- (E) 68

Question 5

To serve 6 people, a certain recipe calls for 2 cups of sugar.
Using this recipe, how many cups of sugar are needed to serve 9 people?

- (A) 3
- (B) 6
- (C) 9
- (D) 18
- (E) 24

Question 6

Julia figures that for every 3 customers entering her flower shop she gets \$20 in sales.
How many customers need to enter her shop for her to get \$300 in sales?

- (A) 2,000
- (B) 900
- (C) 60
- (D) 50
- (E) 45

Question 7

If the ratio of cats to dogs in a certain animal shelter is 5 to 3,
and there are 85 cats, how many dogs are in the pound?

- (A) 6
- (B) 15
- (C) 51
- (D) 93
- (E) 142

Studyguide for the SAT Skill Quiz A: Proportions

Question 8

During the month of May the ratio of grapes to bananas to apples sold in a grocery store was 2:3:5. If the store sold 30 apples in May, how many grapes, bananas, and apples did the store sell in all?

- (A) 30
- (B) 35
- (C) 45
- (D) 50
- (E) 60

Question 9

$1\frac{1}{2}$ is in the same proportion to $1\frac{1}{4}$ as $4\frac{1}{2}$ is to ____?

- (A) 2.5
- (B) 3.75
- (C) 5.4
- (D) 5.8
- (E) 6.2

Question 10

Samuel works in a paint store. Mrs. Adams wants a light blue paint for her bedroom walls. Samuel gets the correct shade of blue by mixing 2 gallons of white paint with 1 gallon of dark blue paint.

How many gallon(s) of dark blue paint is needed to get 5 gallons of the mixture?

- (A) $\frac{2}{5}$
- (B) $\frac{3}{5}$
- (C) $\frac{5}{7}$
- (D) $1\frac{2}{3}$
- (E) $2\frac{1}{2}$

Studyguide for the SAT Skill Quiz A: Proportions



Hint for Question 1:

Cross-multiply and solve for the missing number.

Hint for Question 2:

Cross-multiply and solve for the missing number.

Hint for Question 3:

Create a proportion to solve for the length of the photograph.
Let a variable be the new length.

Hint for Question 4:

Create a proportion expressing centimeters to miles.
Let a variable represent the unknown number of centimeters.

Hint for Question 5:

Create a proportion expressing the number of cups of sugar to the number of people.
Let a variable represent the missing number of cups of sugar.

Hint for Question 6:

Write a proportion expressing the number of customers to sales.

Hint for Question 7:

Write a proportion expressing the number of cats to dogs.

Studyguide for the SAT Skill Quiz A: Proportions

Hint for Question 8:

In this problem you have to find the common factor of the ratios.

2:3:5 is a reduced ratio.

Think of it as $2x:3x:5x$.

If you can find x , you can find the number of grapes,
bananas, and apples.

Hint for Question 9:

Write a proportion to represent the problem.

Be careful handling the fractions.

You may want to write the mixed numbers
as improper fractions.

Hint for Question 10:

Be careful working this problem.

Write a proportion to represent the gallons of dark blue
paint to the total gallons of paint.

How many total gallons of paint are mixed
to get the correct shade of blue?

Studyguide for the SAT Skill Quiz A: Proportions



Answers!



Question 1.

The answer is (C).

Explanation for Question 1:

$$\frac{5}{6} = \frac{15}{x}$$

$$5x = 90$$

$x = 18$. The answer is (C).

Question 2.

The answer is (B).

Explanation for Question 2:

$$\frac{6}{9} = \frac{x}{12}$$

$$72 = 9x$$

$8 = x$. The answer is (B).

Question 3.

The answer is (C).

Explanation for Question 3:

Let x represent the length of the new photograph.

Write the proportion: old dimensions to new dimensions.

$$\frac{\text{old width}}{\text{new width}} = \frac{\text{old length}}{\text{new length}}$$

$$\frac{4}{10} = \frac{6}{x}$$

$$4x = 60$$

$x = 15$. The answer is (C).

Studyguide for the SAT Skill Quiz A: Proportions

Question 4.

The answer is (D).

Explanation for Question 4:

Let x represent the unknown number of centimeters.

Write the proportion expressing centimeters to miles.

$$\frac{2}{5} = \frac{x}{34}$$

Cross-multiply to solve for x .

$$68 = 5x$$

$x = 13.6$. The answer is (D).

Question 5.

The answer is (A).

Explanation for Question 5:

Let x represent the cups of sugar for 9 people.

Write the proportion expressing the number of cups to the number of people.

$$\frac{2}{6} = \frac{x}{9}$$

$$18 = 6x$$

$3 = x$. The answer is (A).

Question 6.

The answer is (E).

Explanation for Question 6:

Let x represent the number of customers needed for \$300 in sales.

Write the proportion expressing the number of customers to sales.

$$\frac{3}{20} = \frac{x}{300}$$

$$900 = 20x$$

$45 = x$. The answer is (E).

Studyguide for the SAT Skill Quiz A: Proportions

Question 7.

The answer is (C).

Explanation for Question 7:

Let x be the number of dogs.

Write a proportion expressing the number of cats to dogs.

$$\frac{5}{3} = \frac{85}{x}$$

$$5x = 255$$

$x = 51$. The answer is (C).

Question 8.

The answer is (E).

Explanation for Question 8:

The ratio 2:3:5 is reduced. Think of the original ratio as $2x:3x:5x$.

This ratio is grapes to bananas to apples. $5x$ represents the number of apples.

In May, 30 apples were sold, so $5x = 30$. $x = 6$. You now know

the common factor for each fruit. Substitute 6 for x in

the ratio $2x:3x:5x$ and get 12:18:30.

The total number of grapes, bananas, and apples sold is the sum of 12, 18, and 30.

The total sold is $12 + 18 + 30 = 60$. The answer is (E).

Studyguide for the SAT Skill Quiz A: Proportions

Question 9.

The answer is (B).

Explanation for Question 9:

Let x be the missing number. The proportion is

$$\frac{1\frac{1}{2}}{\frac{1}{2}} = \frac{4\frac{1}{2}}{x}$$

Change each mixed number to an improper fraction.

$$\frac{3/2}{5/4} = \frac{9/2}{x}$$

Cross-multiply to solve for x .

$$(3/2)x = (9/2)(5/4)$$

$$\frac{3x}{2} = \frac{45}{8}$$

$$x = \frac{45}{8} \times \frac{2}{3} = \frac{90}{24} = 3.75. \text{ The answer is (B).}$$

You could have used decimals instead of fractions.

Studyguide for the SAT Skill Quiz A: Proportions

Question 10.

The answer is (D).

Explanation for Question 10:

The total gallons of paint mixed to get the correct shade of blue is 3 (2 gallons of white plus 1 gallon of dark blue).

Let x represent the gallons of dark blue paint needed to get a 5-gallon mixture.

Write the proportion as gallons of dark blue to total gallons.

The number of gallons of dark blue in the original mixture is 1.

The total number of gallons in the original mixture is 3.

$$\frac{1}{3} = \frac{x}{5}$$

Solve this proportion by cross-multiplying.

$$5 = 3x$$

$$\frac{5}{3} = x$$

$$\frac{5}{3} = 1\frac{2}{3}. \text{ The answer is (D).}$$



CANCEL COMMON TERMS

One skill that will serve you well is the ability to cancel common terms. If you can eliminate common terms from an expression or an equation you will simplify the problem and often quickly deduce the answer.

Methods for canceling common terms rely on multiplying and dividing or adding and subtracting. These techniques are especially useful for Quantitative Comparison problems.

Multiplication or Division

Example 1:
Simplify $\frac{6x^3y^2}{3xy^4}$

Solution:

In the above example, when simplifying, remember to reduce coefficients and subtract exponents of like bases. The largest exponent of the variable dictates if the variable remains in the numerator or denominator.

For the above expression: 6 and 3 are coefficients. They reduce to the value 2. x^3 and x have like bases. Subtract the exponents and the answer will be x^2 in the numerator because the larger exponent is in the numerator. y^2 and y^4 have like bases. Subtract the exponents and the answer will be in the denominator because the larger exponent is in the denominator.

$$\frac{6x^3y^2}{3xy^4} = \frac{2x^2}{y^2}$$

You only can cancel terms that are FACTORS (part of the product) of the whole numerator or whole denominator.

Example 2:

$\frac{xy - c}{x}$ In this problem you CANNOT cancel the x's

because the x in the numerator is not a FACTOR of the whole numerator.

Example 3:

Simplify $\frac{9x^3(x+2)}{12x(x+2)}$

Solution:

In this problem you can do the following:

1. Reduce the coefficients
2. Cancel the x's
3. Cancel the expression $(x + 2)^*$

*Cancel the whole expression NOT the x and 2 individually.

$$\text{The solution is } \frac{3x^2}{4}$$

It is often advantageous to cancel before multiplying.

Example 4:

Simplify the following expression.

$$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9}$$

Solution:

Instead of multiplying the numerator and then the denominator, cancel numerators and denominators.

$$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} = \frac{2}{9}$$

Addition or Subtraction

You also may remove variables by adding or subtracting.

Example 5:

If $x + 9 + y = 10y + x$, what is the value of y?

Solution:

If you subtract x from both sides of the equation, the equation reduces to

$$\begin{aligned} 9 + y &= 10y \\ 9 &= 9y \\ 1 &= y \end{aligned}$$



Question 1

If $x + x + x + x + x = 3x + 10$, what is the value of x ?

- (A) 2
- (B) 5
- (C) 10
- (D) 18
- (E) 20

Question 2

If $-5r = 30$, what is the value of r ?

- (A) -25
- (B) -6
- (C) 6
- (D) 25
- (E) 35

Question 3

If $-8x = -48$, what is the value of x ?

- (A) -40
- (B) -6
- (C) 6
- (D) 40
- (E) 56

Question 4

$$\frac{1}{3y} - \frac{4y}{6y^2} + \frac{6y^2}{9y^3} - \frac{4x}{12xy} = ?$$

(A) $\frac{-(3+4x)}{(3+9y^3+12xy)}$

(B) $\frac{-xy^3}{30xy^7}$

(C) $\frac{1}{3}$

(D) 0

(E) $\frac{2}{y}$

Question 5

Solve for r, if $-7r = -21$.

(A) -3

(B) $-1/3$

(C) 0

(D) $1/3$

(E) 3

Question 6

If $x^2 - y^2 = 15$ and $x + y = 3$, what is the value of $x - y$?

(A) $1/5$

(B) 5

(C) 12

(D) 18

(E) 45

Question 7

Simplify: $(4x^3y^2)/(12xy^3z)$.

- (A) $4x^2yz$
- (B) $1/(3x^2yz)$
- (C) $(3x^2)/(yz)$
- (D) $3x^4y^5z$
- (E) $x^2/(3yz)$

Question 8

If $ax + ax + ax + ax + ax + ax = 2ax - 6x$ and $x \neq 0$, what is the value of a ?

- (A) $-2/3$
- (B) -1
- (C) $-3/2$
- (D) -2
- (E) -3

Question 9

If $x \neq y$, simplify $\frac{y^2 - x^2}{x - y}$.

- (A) $x + y$
- (B) $x - y$
- (C) $y - x$
- (D) $-(x + y)$
- (E) $1/(x + y)$

Grid-in your answer here:

Question 10

If x is a positive integer > 3 , $3x < 2y$, and $2y < 2x + 7$,
what is one possible value for x ?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

**Hint for Question 1:**

Add like terms, get all variables on one side of the equation, and solve for x .

Hint for Question 2:

Divide both sides of the equation by -5 .

Hint for Question 3:

Divide both sides of the equation by -8 .

Hint for Question 4:

Simplify each fraction before trying to find a common denominator.

Hint for Question 5:

Divide both sides of the equation by -7 .

Hint for Question 6:

Factor $x^2 - y^2$ and then divide by $x + y$.

Hint for Question 7:

Cancel the common terms. Be sure to subtract exponents.

Hint for Question 8:

Add like terms, subtract $2ax$ from both sides, and divide to solve for a .

Hint for Question 9:

Factor the numerator and simplify. Be careful of a negative sign.

Hint for Question 10:

How do $3x$ and $2x + 7$ compare? Use this relationship to find a value for x .



Answers!

(A) (B) (C) (D) (E)



Question 1.

The answer is (B).

Explanation for Question 1:

$$5x = 3x + 10$$

$$2x = 10$$

$$x = 5$$

The answer is (B).

Question 2.

The answer is (B).

Explanation for Question 2:

$$\frac{-5r}{-5} = \frac{30}{-5}$$

$$r = -6$$

The answer is (B).

Question 3.

The answer is (C).

Explanation for Question 3:

$$\frac{-8x}{-8} = \frac{-48}{-8}$$

$$x = 6$$

The answer is (C).

Question 4.

The answer is (D).

Explanation for Question 4:

$$\frac{1}{3y} - \frac{4y}{6y^2} + \frac{6y^2}{9y^3} - \frac{4x}{12xy} = \frac{1}{3y} - \frac{2}{3y} + \frac{2}{3y} - \frac{1}{3y} = \frac{0}{3y} = 0$$

The answer is (D).

Question 5.

The answer is (E).

Explanation for Question 5:

$$\frac{-7r}{-7} = \frac{-21}{-7}$$

$$r = 3$$

The answer is (E).

Question 6.

The answer is (B).

Explanation for Question 6:

$$x^2 - y^2 = 15$$

$$(x + y)(x - y) = 15$$

$$\frac{(x + y)(x - y)}{x + y} = \frac{15}{3}$$

$$x - y = 5$$

The answer is (B).

Question 7.

The answer is (E).

Explanation for Question 7:

$$\frac{4x^3y^2}{12xy^3z} = \frac{x^2}{3yz}$$

The answer is (E).

Question 8.

The answer is (C).

Explanation for Question 8:

$$6ax = 2ax - 6x$$

$$4ax = -6x$$

$$\frac{4ax}{4x} = \frac{-6x}{4x}$$

$$a = \frac{-6}{4}$$

$$a = \frac{-3}{2}$$

The answer is (C).

Question 9.

The answer is (D).

Explanation for Question 9:

$$\frac{y^2 - x^2}{x - y} = \frac{(y - x)(y + x)}{x - y} = \frac{-(x - y)(y + x)}{x - y} =$$

$$-(y + x) \text{ or } -(x + y)$$

The answer is (D).

Question 10.

The answer is 4, or 5.

Explanation for Question 10:

From the inequalities you can conclude that $3x < 2x + 7$.

Solving this inequality results in $x < 7$.

x must be a positive integer greater than 3.

So x can be 4, 5, or 6. The answer is 4, 5, or 6.



Question 1

If $x \neq y$, simplify $\frac{x^2 - y^2}{x - y}$.

- (A) $1/(x + y)$
- (B) $x + y$
- (C) $x - y$
- (D) $1/(x - y)$
- (E) $2xy$

Question 2

If $x^2 y^3 = 34$ and $xy^2 = 1/2$, what is the value of xy ?

- (A) $17/2$
- (B) 17
- (C) $67/2$
- (D) 36
- (E) 68

Question 3

If $x^2 + 3xy = 20$ and $x + 3y = 5$, what is the value of x ?

- (A) -15
- (B) 4
- (C) 15
- (D) 25
- (E) 100

Question 4

Solve the inequality: $-10 < 5x < 15$.

- (A) $-10 < x < 5$
- (B) $-2 < x < 15$
- (C) $-5 < x < 20$
- (D) $-2 < x < 3$
- (E) $-15 < x < 10$

Studyguide for the SAT Skill Quiz B: Cancel Common Terms

Question 5

The area of a rectangle is represented by the expression $\frac{7x^2y}{10a^3}$.

If the length of the rectangle is represented by the expression $\frac{14x^3}{25a^2b}$,

which of the following expressions represent the width of the rectangle?

- | | |
|-----------------------|-----------------------|
| (A) $\frac{5ax}{4by}$ | (D) $\frac{5ab}{2xy}$ |
| (B) $\frac{5by}{4ax}$ | (E) $\frac{4ax}{5by}$ |
| (C) $\frac{5xy}{4ab}$ | |

Question 6

If the difference of nine times a number and 7 is equal to the sum of five times the number and 13, what is the number?

- (A) $3/7$
- (B) $1/2$
- (C) 5
- (D) 9.1
- (E) 32

Question 7

If $x^2 + 3xy + 2y^2 = 35$ and $x + 2y = 5$, what is the value of $x + y$?

- (A) $1/7$
- (B) 5
- (C) 7
- (D) 30
- (E) 40

Studyguide for the SAT Skill Quiz B: Cancel Common Terms

Question 8

Which of the following expressions represents the area of a rectangle with

length $\frac{5x^2y}{8ab}$ and width $\frac{12a^2b}{25x}$?

- (A) $\frac{17a^2x^2by}{33abx}$ (D) $\frac{3xy}{10a}$
- (B) $\frac{60ax^2aby}{200abx}$ (E) $\frac{10axy}{3}$
- (C) $\frac{3axy}{10}$

Grid-in your answer here:

Question 9

Evaluate the following: $\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \dots \times \frac{26}{27}$.

<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Grid-in your answer here:

Question 10

If $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} \times \frac{d}{e} \times \frac{e}{f} = \frac{3}{5f}$, what is the value of a ?

<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Studyguide for the SAT Skill Quiz B: Cancel Common Terms



Hint for Question 1:

Factor the numerator and then cancel the common term.

Hint for Question 2:

Divide one equation by the other.

Hint for Question 3:

Factor one equation and divide by the other.

Hint for Question 4:

This is a combined inequality. Divide all parts by 5.

Hint for Question 5:

When you know the area of a rectangle and the length, what operation do you use to find the width?

Hint for Question 6:

Assign a variable to the number. Rewrite these words into an algebraic equation and solve for the variable.

Hint for Question 7:

Factor the first equation and divide by the second equation.

Hint for Question 8:

What do you do with the length and width of a rectangle to find the area of the rectangle?

Hint for Question 9:

Cancel before multiplying.

Hint for Question 10:

Cancel before starting to solve for a.



Answers!

(A) (B) (C) (D) (E)



Question 1.

The answer is (B).

Explanation for Question 1:

$$\frac{(x+y)(x-y)}{x-y} = x+y$$

The answer is (B).

Question 2.

The answer is (E).

Explanation for Question 2:

$$\frac{x^2y^3}{xy^2} = \frac{34}{1/2}$$

$$xy = 34 \times \frac{2}{1} = 68$$

The answer is (E).

Question 3.

The answer is (B).

Explanation for Question 3:

$$\frac{x(x+3y)}{x+3y} = \frac{20}{5}$$

$$x = 4$$

The answer is (B).

Question 4.

The answer is (D).

Explanation for Question 4:

$$\frac{-10}{5} < \frac{5x}{5} < \frac{15}{5}$$

$$-2 < x < 3$$

The answer is (D).

Question 5.

The answer is (B).

Explanation for Question 5:

Divide the area by the length to find the width.

$$\frac{7x^2y}{10a^3} \div \frac{14x^3}{25a^2b} = \frac{7x^2y}{10a^3} \times \frac{25a^2b}{14x^3} = \frac{5by}{4ax}$$

The answer is (B).

Question 6.

The answer is (C).

Explanation for Question 6:

Rewrite the words into algebra. Let n be the number,

$$9n - 7 = 5n + 13$$

$$4n = 20$$

$$n = 5$$

The answer is (C).

Question 7.

The answer is (C).

Explanation for Question 7:

$$x^2 + 3xy + 2y^2 = 35$$

$$(x + y)(x + 2y) = 35$$

$$\frac{(x + y)(x + 2y)}{x + 2y} = \frac{35}{5}$$

$$x + y = 7$$

The answer is (C).

Question 8.

The answer is (C).

Explanation for Question 8:

To find the area of the rectangle you multiply the length and width.

$$\frac{5x^2y}{8ab} \times \frac{12a^2b}{25x} = \frac{3axy}{10}$$

The answer is (C).

Studyguide for the SAT Skill Quiz B: Cancel Common Terms

Question 9.

The answer is .111.

Explanation for Question 9:

$$\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \dots \times \frac{26}{27} = \frac{3}{27} = \frac{1}{9}$$

The answer is .111 or 1/9.

Question 10.

The answer is .6.

Explanation for Question 10:

$$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} \times \frac{d}{e} \times \frac{e}{f} = \frac{3}{5f}$$

$$\frac{a}{f} = \frac{3}{5f} \quad \text{You could cross-multiply or cancel the f's in the denominators.}$$

$$5af = 3f \quad (\text{if you cross-multiply})$$

$$\frac{5af}{5f} = \frac{3f}{5f}$$

$$a = \frac{3}{5}$$

The answer is .6 or 3/5.



QUADRATIC INEQUALITIES

There may be quadratic inequalities to solve on the SAT. Sometimes the solutions to quadratic inequalities can be difficult to understand.

Try to understand the following explanations to the following problems.

The first thing you need to do when you are faced with a quadratic inequality problem is make sure all variables and constants are on the left side of the inequality and 0 is on the right side of the inequality.

Make sure the coefficient of x^2 is positive.

If the coefficient of x^2 is negative you can make it positive by multiplying both sides of the inequality by a negative 1.

Just make sure to reverse the direction of the inequality sign if you multiply by a negative number.

Next factor the quadratic and solve like you would a quadratic equation.

After finding solutions for x you have to examine the inequality sign.

If it is a greater than sign ($>$) or a greater than or equal to sign (\geq), your solution is the area of the number line on the outside of your two x solutions.

If the inequality is a less than sign ($<$) or a less than or equal to sign (\leq), your solution is the area of the number line between the two x solutions.

Example 1:

What is the solution to $x^2 > 3x + 10$?

Studyguide for the SAT Skill Review: Quadratic Inequalities

Solution:

Get 0 on the right side of the inequality.

$$x^2 > 3x + 10$$

$$x^2 - 3x - 10 > 0$$

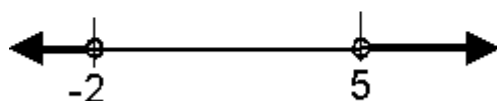
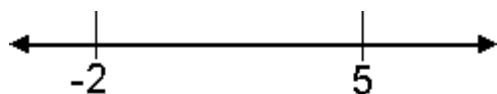
Factor the quadratic.

$$(x - 5)(x + 2) > 0$$

Solve this like you would a quadratic equation.

$$x - 5 = 0 \text{ or } x + 2 = 0$$

$x = 5$ or $x = -2$ Graph these two values on a horizontal number line.



$$x < -2$$

$$x > 5$$

Examine the inequality sign. Because the inequality sign is a greater than sign, the solution area is outside the numbers -2 and 5.

The area outside -2 is $x < -2$ and the area outside 5 is $x > 5$. See the number line above.

The solution to this problem is $x < -2$ or $x > 5$.

Here is the solution to a quadratic inequality using a less than sign.

Example 2:

What is the solution to $x^2 + 8x < -15$?

Studyguide for the SAT Skill Review: Quadratic Inequalities

Solution:

Get 0 on the right side of the inequality.

$$x^2 + 8x < -15$$

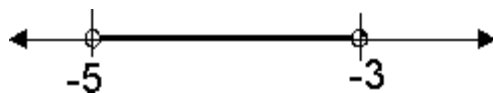
$$x^2 + 8x + 15 < 0$$

Factor the quadratic.

$$(x + 5)(x + 3) < 0$$

Solve for x like you would a quadratic equation.

$$x = -5 \text{ or } x = -3$$



Examine the inequality sign. Because the inequality sign is a less than sign, the solution is the area of the number line between the two solutions for x. Be sure to write the numbers in the correct order. $-5 < x < -3$. The smaller number is to the far left of the combined inequality when using less than signs.

This is a simplified process to solve quadratic inequalities and will work most of the time. If you desire a better explanation as to why this works or want to know how to solve more complicated quadratic inequalities check with your math teacher. The explanations will involve multiplication of positive and negative numbers along with an understanding of intersection and union of sets.



Question 1

What is the solution for $(x - 2)(x + 3) > 0$?

- (A) $x > 2$
- (B) $x < -3$
- (C) $x > 2$ and $x > -3$
- (D) $x > 2$ or $x < -3$
- (E) $x < 2$ or $x > -3$

Question 2

What is the solution for $(x + 2)(x - 3) \leq 0$?

- (A) $-2 \leq x \leq 3$
- (B) $x \geq -2$
- (C) $x \leq -2$ or $x \geq 3$
- (D) $x \leq 3$
- (E) Null Set

Question 3

Solve for x , given $x^2 + 6x > 7$.

- (A) $-7 < x < 1$
- (B) $x > 1$ or $x < -7$
- (C) $-1 < x < 7$
- (D) $x > 7$ or $x < -1$
- (E) $x > -7$

Question 4

What is the solution for $x^2 + 10x + 21 \geq 0$?

- (A) $-7 \leq x \leq -3$
- (B) $3 \leq x \leq 7$
- (C) $x \leq 3$ or $x \geq 7$
- (D) $x \geq -3$ or $x \leq -7$
- (E) $x \geq -3$

Question 5

Solve for $x^2 < 3x + 10$.

- (A) $x < -2$ or $x > 5$
- (B) $x < -5$ or $x > 2$
- (C) $-5 < x < 2$
- (D) $-2 < x < 5$
- (E) Null Set

Question 6

What is the solution for $x^2 - 10x + 25 \geq 0$?

- (A) $x \geq 5$
- (B) $x \leq 5$
- (C) Null Set
- (D) All real numbers, except 0
- (E) All real numbers

Question 7

What is the solution for $3x - 2 > x^2$?

- (A) $-2 < x < -1$
- (B) $x < -2$ or $x > -1$
- (C) $1 < x < 2$
- (D) $x < 1$ or $x > 2$
- (E) $x > -1$

Question 8

Solve for x , given $x^2 + 8x - 20 < 0$.

- (A) $-10 < x < 2$
- (B) $-2 < x < 10$
- (C) $x < -2$ or $x > 10$
- (D) $x < -10$ or $x > 2$
- (E) $x > 10$

Question 9

Solve the inequality, $x^2 - 36 \geq 0$.

- (A) $x \geq 6$
- (B) $-6 \leq x \leq 6$
- (C) $x \geq 6$ or $x \leq -6$
- (D) $x \leq 6$
- (E) $x \leq 6$ and $x \geq 6$

Question 10

Solve the inequality, $4x^2 - 9 < 0$.

- (A) $x > -3/2$
- (B) $-3/2 < x < 3/2$
- (C) $x < -3/2$ or $x > 3/2$
- (D) $x > 3/2$
- (E) $x < -3/2$ and $x > 3/2$

Studyguide for the SAT Skill Quiz A: Quadratic Inequalities



Hint for Question 1:

Because the inequality sign is a greater than sign,
is the solution the area of the number line
outside the x solutions or between the x solutions?

Hint for Question 2:

Because the inequality sign is a less than or equal to sign,
is the solution area of the number line outside the x solutions or between
the x solutions?

Hint for Question 3:

Get 0 on the right side of the inequality.
Is the inequality greater than or less than?

Hint for Question 4:

Factor the quadratic.
Because the inequality is a greater than or equal to sign,
the solution area is outside the x solutions.

Hint for Question 5:

Get 0 on the right side of the inequality.
Because the inequality is a less than sign, the solution area is between
the x solutions.

Hint for Question 6:

Be careful about what this question is asking.
If you factored the quadratic as
 $(x - 5)(x - 5)$ think what it would mean if you wrote the factors as $(x - 5)^2$?

Hint for Question 7:

Get 0 on the right side of the inequality.
Remember to get the coefficient of x^2 positive.
If you distribute a negative number don't forget to
reverse the direction of the inequality sign.

Hint for Question 8:

Factor the quadratic.

The solution area is between the x solutions.

Hint for Question 9:

The solution area will be outside the x solutions.

Hint for Question 10:

The solution area is between the x solutions.



Answers!

(A) (B) (C) (D) (E)



Question 1.

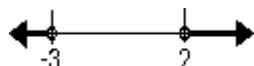
The answer is (D).

Explanation for Question 1:

Because the inequality is a greater than sign, the solution area is outside the x solutions.

$$(x - 2)(x + 3) > 0$$

$$x = 2 \text{ or } x = -3$$



$$x < -3 \text{ or } x > 2$$

The answer is (D).

Question 2.

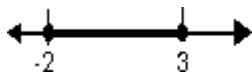
The answer is (A).

Explanation for Question 2:

Because the inequality is a less than or equal to sign, the solution area is between the x solutions.

$$(x + 2)(x - 3) \leq 0$$

$$x = -2 \text{ or } x = 3$$



The solution area is $-2 \leq x \leq 3$.

Because the original inequality was \leq , include \leq in your answer. The answer is (A).

Question 3.

The answer is (B).

Explanation for Question 3:

Get 0 on the right side of the inequality.

$$x^2 + 6x > 7$$

$$x^2 + 6x - 7 > 0 \text{ Factor the inequality.}$$

$$(x + 7)(x - 1) > 0 \text{ Solve for } x.$$

$$x = -7 \text{ or } x = 1$$

Because the inequality is a greater than sign, the solution area is on the outside of the x solutions.

$$x < -7 \text{ or } x > 1$$

The answer is (B).

Question 4.

The answer is (D).

Explanation for Question 4:

$$x^2 + 10x + 21 \geq 0 \text{ Factor the quadratic.}$$

$$(x + 7)(x + 3) \geq 0 \text{ Solve for } x.$$

$$x = -7 \text{ or } x = -3$$

Because the inequality is a greater than or equal to sign, the solution area is outside of the x solutions.

$$x \geq -3 \text{ or } x \leq -7 \text{ The answer is (D).}$$

Question 5.

The answer is (D).

Explanation for Question 5:

$$x^2 < 3x + 10$$

$$x^2 - 3x - 10 < 0 \text{ Factor the quadratic.}$$

$$(x - 5)(x + 2) < 0 \text{ Solve for } x.$$

$$x = 5 \text{ or } x = -2$$

The solution area is between the two x solutions.

$$-2 < x < 5. \text{ The answer is (D).}$$

Studyguide for the SAT Skill Quiz A: Quadratic Inequalities

Question 6.

The answer is (E).

Explanation for Question 6:

$$x^2 - 10x + 25 \geq 0 \text{ Factor the quadratic.}$$

$$(x - 5)^2 \geq 0 \text{ Stop and think. } \geq 0 \text{ means the answer is positive or equal to 0.}$$

When you square a number isn't it always positive or zero.

No matter what you substitute for x the solution for $(x - 5)^2$ will be positive or zero.

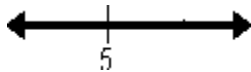
The solution set is all real numbers. The answer is (E).

Another way to get the solution follows.

$$(x - 5)(x - 5) \geq 0. \text{ Solve for } x.$$

$x = 5$ or $x = 5$ Because the inequality is a greater than or equal to sign, the solution area is outside the x solutions.

$x \leq 5$ or $x \geq 5$. Look at this graph.



The graph is the whole number line.

The number line represents all real numbers.

Question 7.

The answer is (C).

Explanation for Question 7:

$$3x - 2 > x^2 \text{ Get 0 on the right side of the inequality.}$$

$$-x^2 + 3x - 2 > 0 \text{ Make the coefficient of } x^2 \text{ positive.}$$

$$x^2 - 3x + 2 < 0 \text{ Reverse the direction of the inequality sign. Factor the quadratic.}$$

$$(x - 1)(x - 2) < 0 \text{ Solve for } x.$$

$$x = 1 \text{ or } x = 2.$$

Because the final inequality sign is a less than sign, the solution area is between 1 and 2.

$$1 < x < 2. \text{ The answer is (C).}$$

Question 8.

The answer is (A).

Explanation for Question 8:

$$x^2 + 8x - 20 < 0 \text{ Factor the quadratic.}$$

$$(x + 10)(x - 2) < 0 \text{ Solve for } x.$$

$$x = -10 \text{ or } x = 2 \text{ The solution area is between } -10 \text{ and } 2.$$

$$-10 < x < 2. \text{ The answer is (A).}$$

Question 9.

The answer is (C).

Explanation for Question 9:

$$x^2 - 36 \geq 0 \text{ Factor the quadratic.}$$

$$(x - 6)(x + 6) \geq 0 \text{ Solve for } x.$$

$$x = 6 \text{ or } x = -6. \text{ The solution area is outside of the } x \text{ solutions.}$$

$$x \leq -6 \text{ or } x \geq 6. \text{ The answer is (C).}$$

Question 10.

The answer is (B).

Explanation for Question 10:

$$4x^2 - 9 < 0 \text{ Factor the quadratic.}$$

$$(2x - 3)(2x + 3) < 0 \text{ Solve for } x.$$

$$2x = 3 \text{ or } 2x = -3$$

$$x = 3/2 \text{ or } x = -3/2 \text{ The solution area is between } 3/2 \text{ and } -3/2.$$

$$-3/2 < x < 3/2. \text{ The answer is (B).}$$